GROUNDWATER RESOURCE EVALUATION THROUGH MODELLING IN AN AREA WITHIN THE KRISHNA RIVER BASIN, ANDHRA PRADESH

A Thesis Submitted
In Partial fulfilment of the Requirements
for the Degree of

DOCTOR OF PHILOSOPHY

by

M. V. R. L. MURTHY

to the

DEPARTMENT OF CIVIL ENGINEERING

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Post Graduate Office
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CERTIFICATE

Certified that this work, " GROUNDWATER RESOURCE EVALUATION THROUGH MODELLING IN AN AREA WITHIN THE KRISHNA RIVER BASIN, ANDHRA PRADESH " by M.V.R.L. MURTHY has been carried out under my supervision and that this has not been submitted else where for a degree.

Feb.28. 1989

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M V R L MURTHY

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SYNOPSIS

In India since the advent of river valley projects, irrigation through canal systems has brought about significant changes in the groundwater levels in the aquifers in canal command areas. These changes in hard-rock areas are so pronounced that drastic rise in water levels over the years has resulted in some localities through excess seepage from canals and less abstraction from wells. Thus, some of the regions in hard-rock terrains, once drought-prone, are

currently facing or likely to face the threat of water-logging within the canal command areas. The consequences of the dominant seepage becomes apparent with time. The aquifer behaviour in such situations can be well understood, if all the inflows and outflows involved in a system are considered. The complex nature of the interaction between such features necessitates the use of mathematical modelling of aquifer behaviour. Once all the inputs to the model are available, any of the parameters can be established using either the direct problem or the inverse problem. For example, the spatial and temporal distribution of water table heads can be worked out through direct problem using all other relevant input parameters. Similarly, in hard-rock areas, where information on aquifer parameters is scarce due to inadequate well testing facilities and procedures, estimation of these parameters can be done by modelling using inverse problem when all other system elements including the spatial and temporal distribution of water table heads are available. If the information on all the system elements is known in a canal command area, it is not only possible to predict the water-logging at different places but also to suggest corrective measures through proper additional abstractions etc. The topic for the present study has thus been chosen keeping in view the problems of a canal command area.

The study area is a part of the Krishna river basin and situated within the Nagarjuna Sagar left canal command area in

Andhra Pradesh. Covering 830 sq. km, this area has the Nagarjuna Sagar left canal in the north and three rivers (Musi on west, Paleru on east, and Krishna on south) as its boundaries. Irrigation is carried for 9 months in a year through a network of canals and abstraction through groundwater. Since the inception of the canals, regular monitoring is being done on the rainfall, water table heads, canal seepage and evaporation within the area and historical data are available for the same with the A.P. State Groundwater and Irrigation Departments. In the present work data pertaining to a period of ten years (1978 to 1988) at 69 observation wells randomly distributed in the area were used for the modelling. The area has been divided into a 20 X 20 matrix with a grid size of 2.5X2.5 km^2 and all the observation points are located on the same. The water table head data were processed to remove noise and the second spatial as well as the first temporal derivatives were calculated. These values together with the values for the well draft, irrigation effect and rainfall recharge coefficient were assigned at each nodal point as inputs to a discretised Boussinesq's equation and the aquifer parameters (transmissivity and storativity) are calculated for the entire period (1978 to 1988) in two blocks of five-year periods each with the help of a non-linear optimization routine. To evaluate the accuracy of the model, the water table head values were back-calculated using the estimated aquifer parameter values at each of the observation points. The deviation of the estimated head values from the

actual head values was explained on the basis of the assumptions in the calculation of inputs as also the causative geological features. The accuracy of the model has also been tested by split sampling method. In addition, prediction of the changes in the water table situation at the end of next five year period (1993) was carried out using this model by incorporating a drainage channel and additional abstraction of ground water through wells. This pilot study was taken up as an illustration of the application of the model for water-logging problems.

The work presented in the thesis is organized in six chapters. In Chapter 1, the topic has been introduced and details of the study area are given. The geological status of the area has also been presented. Unclassified granites of Archean age occurring in northern and central parts of the area are the dominant formations covering around 70% of the terrain. A few dolerite dykes with a general NE-SW trend are present in the western half of the granitic terrain. The granites are in turn overlain by quartz arenites and limestones, both belonging to Kurnool Super group (Upper Proterozoic in age). A few lineaments and two faults located on the basis of satellite imageries conform in their directions to the regional tectonic trend. The groundwater situation in the study area is described in this chapter and typical well hydrographs for all the three formations are presented. The scope of the present study has also been

briefly dealt with. In Chapter 2, a review of modelling techniques for groundwater resource evaluation has been presented. The inverse and direct problems have been introduced. As the main emphasis was on the inverse problem in the present study, different methods of solving the inverse problem are presented with special reference to equation error criterion and the output error criterion.

The processing of water table data forms the theme for Chapter 3. The relevant details of the methodology for water table head reconstruction using least square polynomial approximation are presented. The head data for sixty months (1978 to 1983) for all the 69 observation wells have been processed. The coefficients of the ploynomial fitted for this data have been calculated by considering an optimal form of a fourth degree polynomial equation which has been achieved by minimizing the standard error on considering a third degree polynomial. The decision for the same is made on the basis of students' t test. The goodness of fit of the estimated coefficients has been checked through a multiple correlation coefficient. The estimated polynomial coefficients were utilized in computing the second spatial and first temporal derivatives, which were used later for the estimation of aquifer parameters. Typical water table contour maps and trend surface plots have also been presented. Effects of dykes and the geological structures (lineaments and faults) on the water table configuration have been discussed.

In Chapter 4, the recharge and discharge components for the study area have been assigned for 120 months and at 69 observation wells. A coefficient of 0.15 for recharge due to rainfall has been taken to be characteristic of the area. Based on the monthly evaporation rate obtained from A.P.State Ground Water Department, losses due to evaporation were calculated. Recharge due to canal seepage has been estimated by considering conveyance losses for both the main and branch canals. 5% of the canal discharge has been considered to be irrigation effect.

Estimation of aquifer parameters has been carried out using the inverse problem approach. The methodology along with the relevant mathematical background is presented in Chapter 5. The Boussinesq's equation was discretised with respect to system elements (transmissivity in X and Y directions, storativity, angle between the principal permeability directions and rainfall recharge coefficients). Taking the recharge and discharge parameters in a discretised form and derivatives of the water table head, an objective function has been formulated to estimate the system parameters. This function was solved by minimizing the sum of the squares of the residual errors with the help of a non-linear optimization routine (Sequential Unconstrained Minimization Technique). The optimal values of the system parameters for both the five-year blocks (1978-1983 and 1983 to 1988) were estimated when the error criterion is satisfied. The transmissivity estimates are

around 50 to 60 m^2/day with a variation upto 430 m^2/day in areas with dykes, lineaments and faults. The storativity estimates for the area are around 0.160, characteristic of unconfined aquifers. The angle between the principal permeability directions was estimated as 0.78 radians and the estimated values of recharge coefficient due to rainfall varied from 0.132 to 0.187. The groundwater storage has also been estimated for the area around each of the observation points.

The accuracy of the model has been checked by simulating the head values, the estimated parameters and other related inputs using Alternating Direction Implicit Scheme. The deviation between the observed and estimated head values at different places was interpreted in terms of the geological factors such as the dykes and the assumption in the calculation of rainfall recharge. Effect of some of these factors has also been confirmed through use of split sampling procedure by taking estimated transmissivity values along with the recharge coefficients obtained for all the 69 observation points at the end of first five- year period as inputs for the estimation of heads for the second five-year period. In general, the error has been found to be less than 7% with 62 out of 69 points having an error less than 5% . The values of transmissivity for all the 69 observation points for the two five-year periods along with the groundwater storage values have been presented in this chapter.

The summary of the findings in terms of a comprehensive integrated picture is presented in Chapter 6. Application of the model for the study of water-logging problems has been illustrated. For this purpose, the groundwater configuration at the end of December 1987 has been chosen and water-logged areas have been identified. Incorporating a drainage channel and additional drafts through wells, the aquifer behaviour has been simulated using the Alternating Direction Implicit Scheme. A reduction of water table up to 3 m in the water-logged areas has been observed in the predicted head distribution for the period ending May 1993.

CHAPTER 1

INTRODUCTION

1.1 GENERAL

With the inception of Nagarjuna Sagar project in Andhra Pradesh in 1966, considerable change has come about in the groundwater regime within the canal command area. Canal irrigation has resulted in pronounced rise of the groundwater table at several places in hard-rock terrain which has hitherto been drought-prone with deep water table conditions. After two decades of irrigation practice, high water table has created water-logging in several zones in the left and right canal command areas.

Prediction of aquifer behaviour as related to surface resources can effectively be carried out through modelling. In the present work, a part of the Nagarjuna Sagar left canal command area has been taken up for study to evaluate the groundwater situation. Data over a period of ten years (1978 to 1988) pertaining to various inflows and outflows and water table heads were utilized to estimate the aquifer parameters, which in turn have been used to work out the groundwater heads at different periods of time. The model has also been utilized for predicting changes in groundwater table heads by

simulating additional draft through wells in areas of high water table through an additional drainage channel incorporated in the model.

1.2 THE STUDY AREA

1.2.1 Location and Land Use

The area chosen for study is situated in Nalgonda district of Andhra Pradesh and covers 830 sq.km with a maximum length and width of 45km and 60km respectively. Confined within latitudes $16^{\circ}38'$N to $17^{\circ}13'$N and longitudes $79^{\circ}38'$E to $80^{\circ}08'$E, the area has a general gradient of 1.75m/km, from north to south with highest and lowest elevations of 140m and 60m respectively. With a good net work of fair weather roads providing accessibility to all villages, it is also well connected to the two prominent cities Vijayawada and Hyderabad. Of the total area of 830 sq.km of the basin, about 500 sq.km is under cultivation. The rest of the area is generally barren and has scanty vegetation. The basin as a whole is deficient in rainfall. The area is irrigated with both canal water and groundwater for 9 months in a year. Rice cultivation is mainly restricted to areas adjacent to canals. In addition to rice, pulses and groundnut are the important crops raised (Table 1.1). On the whole, irrigated area is about 60 percent of the total area of the basin.

1.2.2 Hydrometeorology

The basin has two rain gauge stations located in Huzurnagar and Kodad towns (Fig.1.1). The data collected from

Table 1.1 : Cropping Pattern in the Study Area

crop	area (%)
Rice	51.00
Groundnut	15.20
Chillies	4.42
Cotton	3.53
Millets	0.80
Vegetables	10.70
Pulses and Oil seeds	13.35
Sugar cane	1.00

these two places are utilized in computing the water budget. In addition, the evaporation data used in the study was collected from the climatological station located at Huzurnagar. The annual evaporation is around 1800 mm. Based on the point rainfall measurements collected from the two rain gauge stations, annual rainfall distribution graph is prepared for the years 1978-1988 (Fig.1.2).

1.2.3 Surface Hydrology

The study basin is bounded by three rivers and a canal. The Nagarjuna Sagar Left Canal (NSLC) which forms the northern boundary was commissioned in 1966, takes off from a dam on Krishna river at Nagarjuna Sagar with its flow from west to east. This canal with a length of 37km in the study area has several branch canals traversing the region. The hydraulic particulars of main canal are presented in Table 1.2. The area irrigated under each branch canal is shown in Table 1.3.

The Paleru river on the eastern margin is controlled by canal supplies. The paleru lake at the extreme north of the basin is connected with a regulator to NSLC main canal and Paleru river. This lake acts as a balancing reservoir for the whole study area. The Musi river on the western boundary has meagre flow and is filled with silt for maximum period in a year. This river has its confluence in the south with Krishna riverwhich has its flow from east to west and acts as a discharging boundary for the study region.

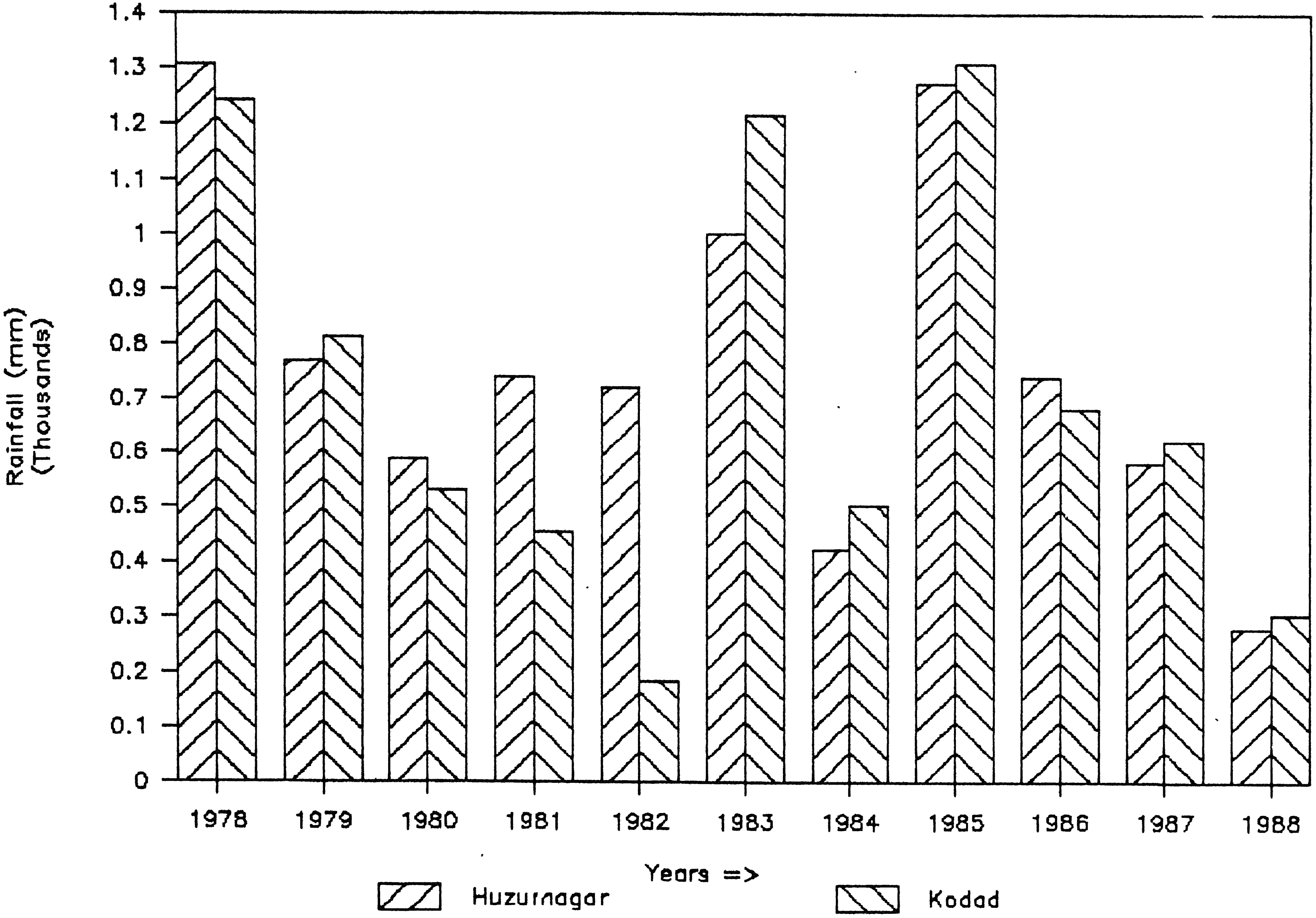


Fig. 1.2 Rainfall distribution in the study area.

Table 1.2* : Hydraulic Particulars of Left Main Canal Within the Study Area

sl.no.	block no.	length (km)	availability of water (cumecs)
1.	8-11	9.850	225.17
2.	11-13(P)	7.400	169.46
3.	13(P)-15	20.945	157.60

Table 1.3* : Hydraulic Particulars of Branch Canals

sl. no.	name of canal	block no.	length (km)	discharge (cumecs)	area irrigated (m^2)
1.	Nagulapadu	8	3.04	0.3925	35226.72
2.	Annaram-I	9	0.33	0.1064	817813.77
3.	Annaram-II	9	1.25	0.3579	3186234.8
4.	Somavaram	9	4.38	1.0113	9251012.2
5.	Chillapalli-I	9	9.47	1.8501	16943320
6.	Chillapalli-II	10	7.55	1.5322	13805668
7.	Jonapadu	10	24.00	15.7221	134793000
8.	A.R.Gudem	10	9.11	1.4791	13526316
9.	Ponugodu	11	2.27	0.3534	2834008
10.	Sarvaram	11	8.74	2.7012	24375789
11.	Kothaguda	11	1.34	0.1192	1093117.4
12.	Kalavapalli	12	8.51	2.0945	18704453
13.	J.P.Gudem	12	3.59	0.6567	5886639
14.	Baithavole	13	3.62	1.0294	8910931
15.	Barakatgudem	13	1.20	0.3415	1720648
16.	13-D	13	1.50	0.0872	858299
17.	Mutyala	13	29.41	49.5829	71457000
18.	Komarabanda	13	8.61	1.1475	9943320
19.	Polavaram	14	12.89	2.3570	3522267
20.	Kotthagudem	14	26.81	7.6563	67797571
21.	Raju Pet	15	3.89	0.7717	20109312

* Source of data: A.P. State Irrigation Dapartment.

1.3 GEOLOGICAL SET UP AND GROUNDWATER SITUATION

1.3.1 Geology

A substantial portion (about 70 percent) of the study area has unclassified crystalline outcrops of Archean age composed mainly of granite with small pockets of hornblende- and quartz-schists. These are succeeded by the quartz arenites (of the Banganapalle group) which are in turn overlain by the Narji limestones (of the Jammalamadugu group), both belonging to the Kurnool Super Group (Fig.1.1). The geological succession is as follows:

Upper Proterozoic	Kunool Super Group	Narji limestones Banganapalle quartz arenites
------------- Unconformity -------------		
Archean	Unclassified Crystallines	Granites with basic intrusives and schists

The granites are basically of two types-the older grey type followed by the younger porphyritic type. The grey series have lithological variation to pink granite. The porphyritic division occurs as intrusive laccolithic masses and dyke-like injections in the former type. The granites in the central region are highly weathered. Erosion along weak zones is quite prominent in the area south of Nagarjuna Sagar left canal and NE of Kamalacheruvu. Fine-grained dolerite dykes occur in granite and these are most important in controlling groundwater condition of the study area. The strike of

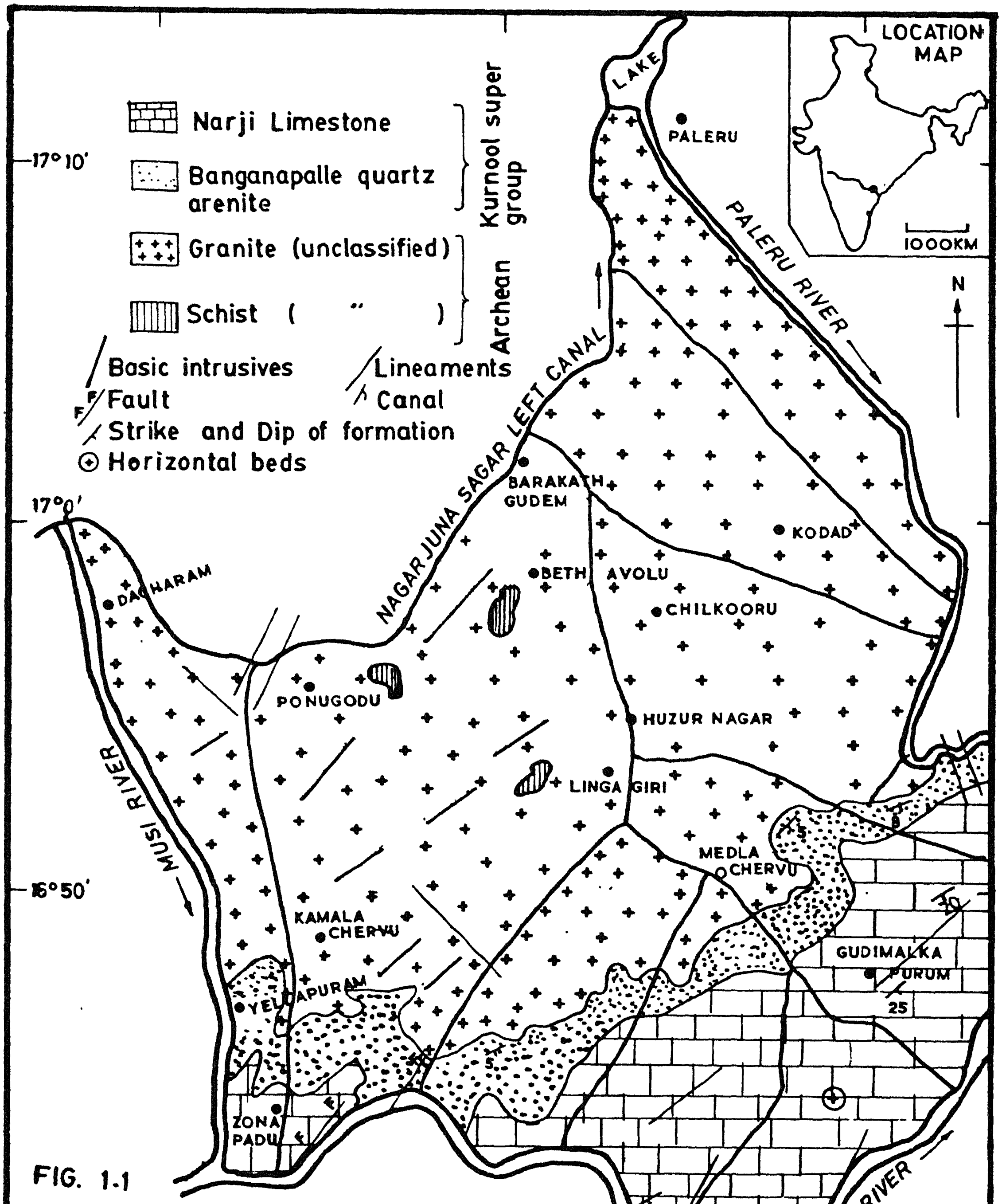


FIG. 1.1

GEOLOGICAL MAP OF THE STUDY AREA

dolerite dykes varies from NE-SW to NNE-SSW. These dykes follow prominent weak zone directions. The associated schists vary in composition from a quartz-schist to hornblende-schist. Ferrugeneous quartz-schists occurring as a patch SW of Betahvolu village have a trend N295° with foliation dipping towards NE. The quartz-hornblende schists located west of Lingagiri have a general strike of N320° with foliation dipping towards ENE. These schists are traversed by muscovite-bearing quartz veins. Details of lithology of formations encountered in a bore hole upto a depth of about 70m in the Kodad area are indicated in Table 1.4.

The sedimentary formations in the northeastern and southern parts of the area constitute a part of the Palanad sub-basin on the extreme northern part of the Cuddapah Basin. The quartz arenites belong to the Banganapalle group and are coarse to medium grained with ferrugeneous matrix. The general strike of the formations is around N 80°-85° with low dips (5° to 8°) towards south. These formations exhibit current bedding of trough type with frequent changes in current directions as evidenced in some exposures, indicative of their deposition in a shallow transgressive environment. The Narji limestone formations of Jammalamadugu stage occuring in the study area are located on the northern border of the Krishna river. These are represented by massive fine-grained limestones with occasional argillaceous pockets. The limestones are siliceous and have a NE-SW strike with varying dips towards south. At places the dips are very low in amount (about 5°) and the formations are horizontally bedded in the area NE of

Table 1.4 : Details of Drill Hole Log at Kodad Town.
(based on data form A.P.S.G.W.Department,A.P)

Formation	Depth (m)	Thickness(m)
Top surface soil,laterite fine to medium-grained	upto 3.0	3.0
Granite, fresh, pink and grey with negligible weathering and fractures at 8.10 and 8.90 m	3.0 to 9.0	6.0
Granite, pink with grey feldspar, weathered and fractured with clay fillings	9.0 to 11.0	2.0
Granite, grey with pink feldspars and mafics, hard with negligible weathering, fractures at 18.8 and 20.0 m	11.0 to 24.0	13.0
Granite, pink and grey, medium hard fresh but with fractures at 36.5 m	24.0 to 51.0	12.0
Dolerite, brittle and fine grained	51.0 to 70.0	19.0

Chitryala.

The change in the course of the Krishna river at places in the study area appears to have been controlled by structural features. For example, the trend of the river in the northeastern part corresponds to the lineament (trending NNE-SSW) observed in the satellite imagery. Similarly the change in the river bend, after its confluence with Musi river, coincides with one of the two faults indicated in the geological map of the Cuddapah Basin (Geological Survey of India, 1981). These faults trending NE-SW extend into the study area (Fig.1.1), one of them traversing the limestone while the other occurring in the quartz arenite formation. While extensive fracturing and disturbance is evidenced within these formations in the vicinity of the faults, the dislocation is not seen in the field since these are intraformational faults and the outcrops are disturbed. The lineaments, as observed on the satellite imagery, have NE-SW trend. It is interesting to note that the trend of these lineaments is similar in direction to that of the dykes present in the study area, possibly conforming to the regional tectonic trends. The tectonic framework of the region south of the study area is reported by Drury and Holt (1980).

1.3.2 Groundwater Occurrence and Aquifer Characteristics

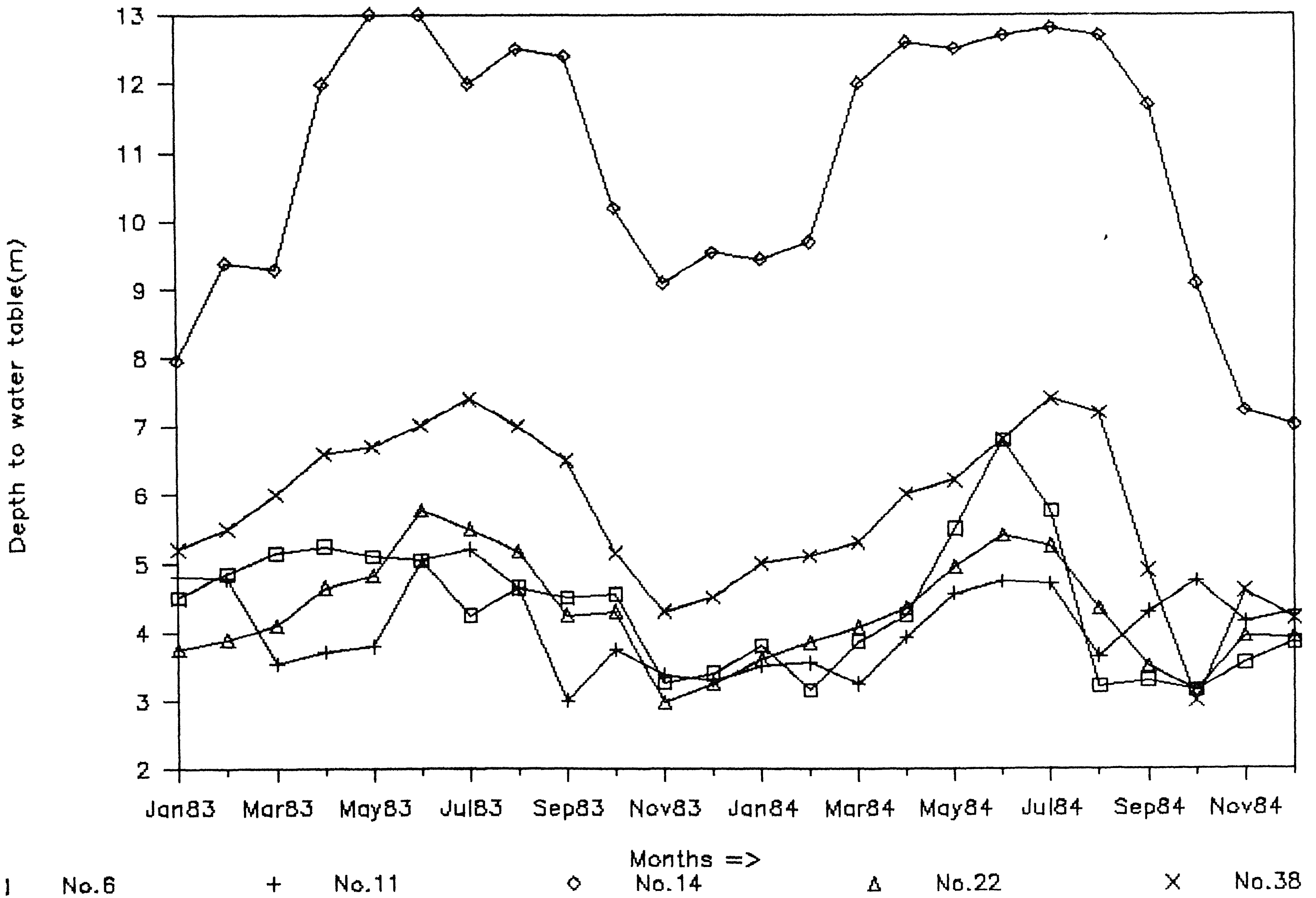
Groundwater occurs under unconfined conditions. Weathered and fractured horizons constitute the aquifer zone. The wells in the area are both of large diameter type and tube wells. Their depths vary in different formations. In the granites and

quartz arenites the wells are upto 13m while in the limestone area, the depth varies up to 22m. The water table head varies from around 144m in the north to around 54m in the south east corner of the study area. Typical hydrographs for ~~for~~ a period of 2 years (Jan.84 to Dec.85) for selected wells in all the three formations are presented in Figs. 1.3, 1.4 and 1.5. Transmissivity (T) and storativity (S) values for the granites as reported by the Andhra Pradesh State Groundwater department are in the range of 60-170m^2/day and 0.2-0.24 respectively. No information is available on the aquifer parameters of the sedimentary formations in the study region.

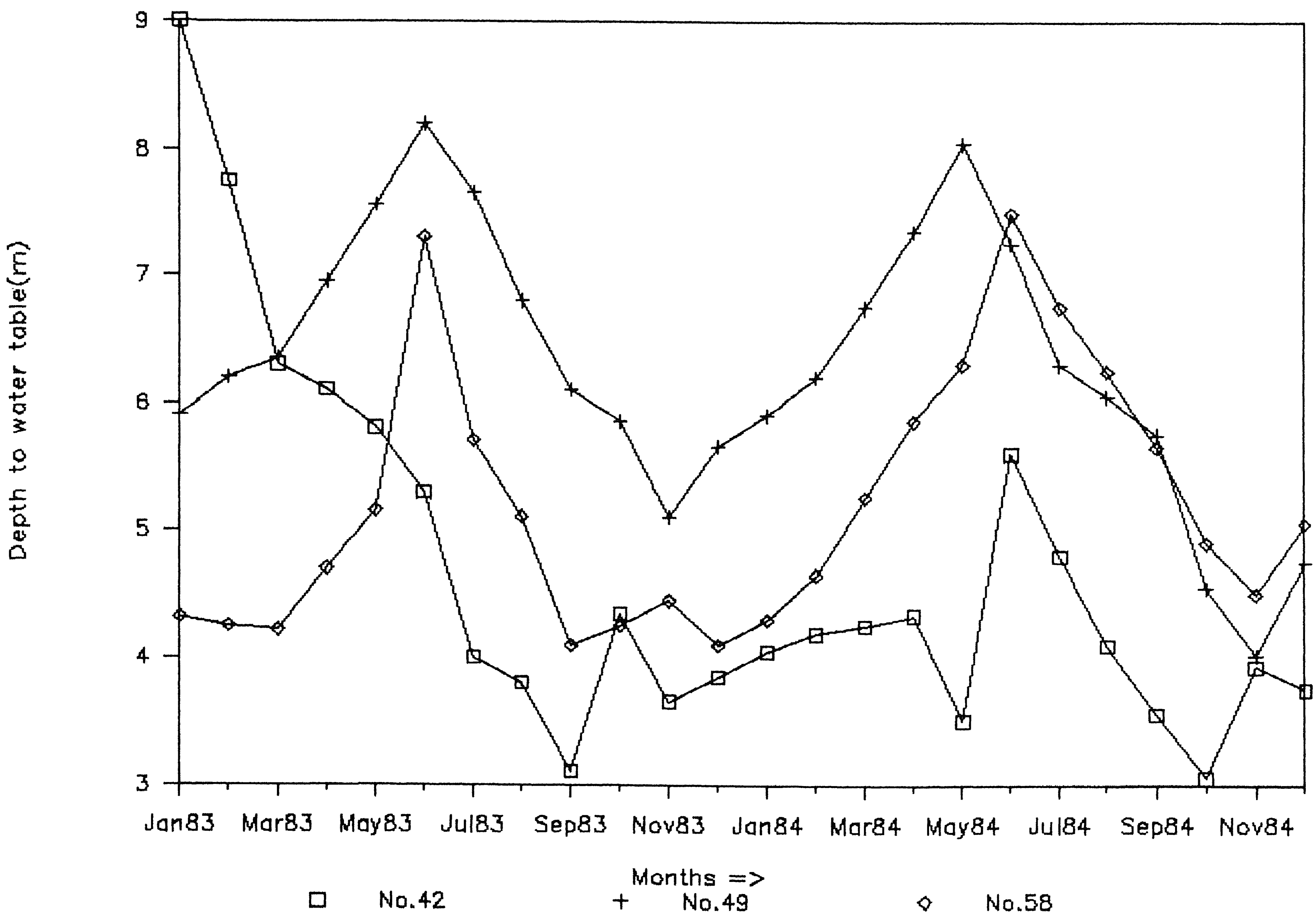
1.4 SCOPE OF THE PRESENT STUDY AND ITS ORGANIZATION

The groundwater regime in an area is controlled by several parameters such as recharge from rainfall and canal seepages, aquifer characteristics, groundwater draft and flows across the boundary. A proper understanding of the interplay of all these parameters can be achieved through modelling. Modelling forms an integral part of groundwater investigations. In the present work a portion of left canal command area of Nagarjuna Sagar project in Andhra Pradesh has been chosen for the study. Surrounded by three rivers and a canal as its boundaries on four sides, the area has been irrigated through canal irrigation for over two decades.

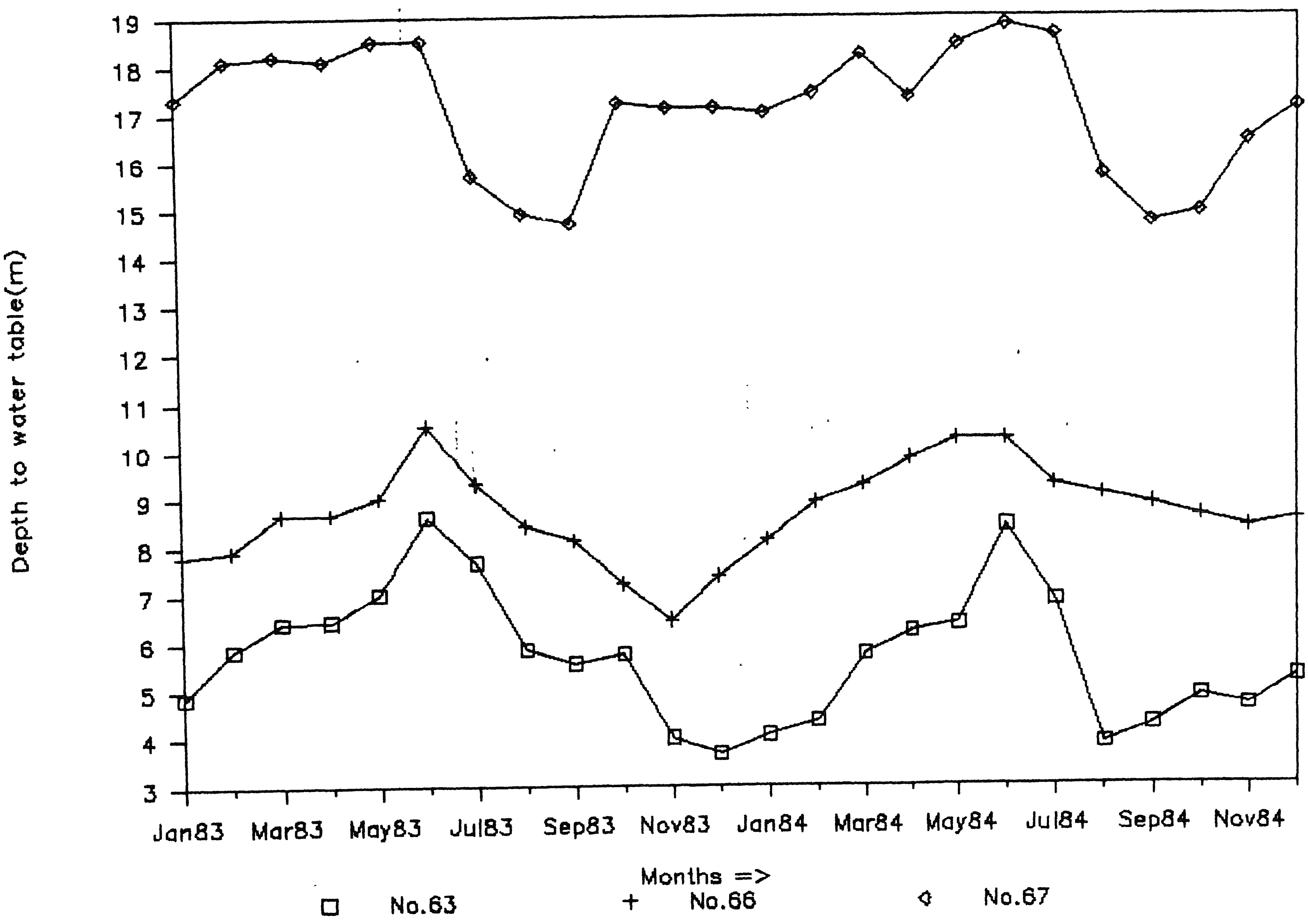
The data for a period of ten years between June 1978 to May 1988 pertaining to various components involved have been utilized in the model. Processed water table head values after



1.3 Well Hydrographs in the Granitic Region



1.4 Well Hydrographs in the Quartz Arenite Region



1.5 Well Hydrographs in the Limestone Region

removing the noise in the data have been used together with the other calculated recharge and discharge values as inputs and the aquifer parameters (transmissivity and storativity) have been estimated using the inverse method. These estimates were in turn utilized for checking the validity of the model through direct problem approach by simulating the water table head data, which were compared with observed field data. The model has also been applied to monitor the water table changes with additional well drafts incorporated.

The material contained in the thesis is organized and presented under several chapters.

In Chapter 1 the study area is introduced in terms of its location, land use, meteorological background, geology and groundwater occurrence. The scope of the present study is also outlined.

In Chapter 2, a review of modelling techniques for groundwater resource evaluation has been presented. The inverse and direct problems have been introduced and their scope indicated.

The processing of water table data forms the theme of Chapter 3. The relevant mathematical background for water table head reconstruction using least square polynomial approximation has been presented in detail. The processing of water table data for the study area has been presented. Two-dimensional (contour maps) and three-dimensional (trend surface maps) graphical representation of the water table head variation for different periods of time are presented.

Estimation of recharge and discharge components and the recharge evaluation for the rainfall data together with the estimation of groundwater system elements are developed in Chapter 4. The recharge and discharge components for the study area are estimated for 120 months and at 69 observation points.

Chapter 5 deals with estimation of aquifer parameters using the inverse problem approach. The methodology with relevant mathematical background pertaining to discretisation of governing equation, estimation of derivatives and optimization has been presented. The aquifer parameters and the groundwater storage have been worked out for the study area. In addition, the rainfall recharge coefficients and the angle between principal permeability directions were obtained. The calculated aquifer parameters have been used for testing the validity of the model. Estimation of water table heads has been carried out by simulation and the comparison of the estimated values with the observed data has been attempted to analyze the error range. The anomalous values of the water table heads have been interpreted in terms of basin characteristics.

An integrated picture has been presented in Chapter 6. The application of the model for monitoring the groundwater variation with additional well draft in areas of shallow water table has been also illustrated by simulating the situation for a particular set of values.

CHAPTER 2

GROUNDWATER RESOURCE EVALUATION BY MATHEMATICAL MODELLING - A REVIEW

2.1 INTRODUCTION

Significant research work has been conducted in the field of groundwater modelling in the last three decades. It is used extensively in understanding aquifer flow, as well as in evaluating regional groundwater resources. If adequate data is available, it is an ideal tool for development of basin-wise plan. A review is attempted in the present chapter to bring out the main applications with emphasis on inverse problem pertinent to the present study.

2.2 MODELLING TECHNIQUES

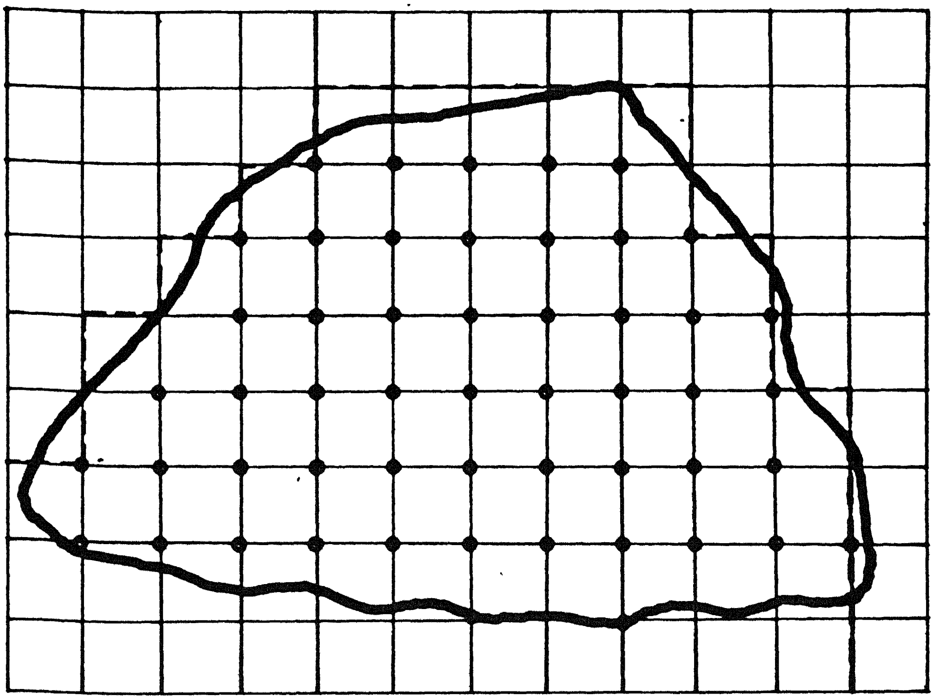
Model is a tool designed to represent a simplified version of reality (Wang and Anderson, 1982). A mathematical model consists of a set of linear differential equations that are known to govern the flow of groundwater. Mathematical models of ground water flow have been in use since the beginning of this century. The reliability of predictions using groundwater model depends on how well the model approximates the field situations. Since the field situations

are too complicated to be simulated exactly, assumptions must always be made in order to construct a model. Usually such assumptions necessary to solve a model analytically are restrictive.

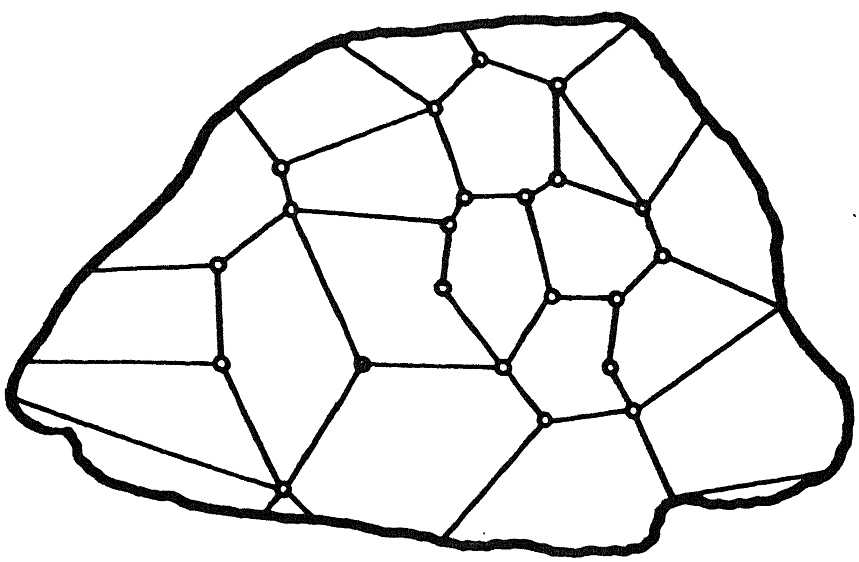
One can consider two types of models-finite difference models and finite element models. In either case, a system of nodal points are superimposed over the problem domain. In the finite difference models, nodes are located either at the centre of the grid lines as in a rectangular grid pattern (Fig.2.1a) or inside the cell as in polygonal grid pattern (Fig.2.1b). While in the former case mesh-centered nodes are used, block- centered nodes are in use in the latter. The concept of elements is fundamental to the development of equations in the finite element method.

In finite difference modelling, the governing equation of flow is approximated using finite differences and resulting set of linear or nonlinear algebraic equations is solved by using direct or iterative techniques. Stallman (1956) was the first to apply numerical methods to groundwater hydrology. Aquifer problems in which transmissivity is independent of variations of the piezometric head are linear problems. Thus confined and leaky aquifers generally lead to linear problems while unconfined aquifer problems may be nonlinear.

For unconfined aquifers, the solutions are greatly facilitated if Dupit-Forchheimer's assumptions are considered. In such a case, the governing equation (Remson et al,1971) for two-dimensional transient groundwater flow in an isotropic, homogeneous, unconfined aquifer, with no vertical accretion is



(a) Rectangular grid



(b) Polygonal grid

FIG. 2.1 GRID PATTERNS FOR FINITE DIFFERENCE MODEL

as follows

$$K \left[\frac{\partial}{\partial x} \left(h \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(h \frac{\partial h}{\partial y} \right) \right] = S \frac{\partial h}{\partial t} \quad \ldots\ldots (2.1)$$

where K is the permeability, h is the hydraulic head, x and y represent distances in the respective directions of flow, t is the time increment and S is the specific yield. Taking into consideration the anisotropy, nonhomogeneity, and the vertical accretion, Eqn.2.1 has been modified (Hollond and Dracup, 1970) as

$$\frac{\partial}{\partial x} \left[K_{xx} h \frac{\partial h}{\partial x} \right] + \frac{\partial}{\partial y} \left[K_{yy} h \frac{\partial h}{\partial y} \right] + Q = S \frac{\partial h}{\partial t} \quad \ldots\ldots (2.2)$$

where x and y refer to the principal permeability directions, and K_{xx} and K_{yy} are the permeabilties in directions x and y respectively and Q is the net inflows and outflows with change in storage.

The governing differential equation (Rushton and Rathod, 1985) for three-dimensional time-variant flow in an aquifer is

$$\frac{\partial}{\partial x}\left[K_x \frac{\partial \phi}{\partial x} \right] + \frac{\partial}{\partial y} \left[K_y \frac{\partial \phi}{\partial y} \right] + \frac{\partial}{\partial z} \left[K_z \frac{\partial \phi}{\partial z} \right] = S_s \frac{\partial \phi}{\partial t} \quad \ldots\ldots (2.3.a)$$

where K_x, K_y and K_z are the hydraulic conductivities in the coordinate directions, and S_s is the specific storage, and

$$\phi \left[x,y,z,t \right] = z + \frac{p(x,y,z,t)}{\rho g} \qquad ..... (2.3.b)$$

in which z is the height above an arbitrary datum, p(x,y,z,t) is the pressure, ρ is the density of the fluid and g is the acceleration due to gravity.

2.3 AQUIFER RESPONSE

The water balance equation proposed by Sokolov and Chapman (1974) can be written as

$$\text{Inflows} - \text{Outflows} - \text{Increase in storage} + \text{Error term} = 0 \qquad ...(2.4)$$

Inflows include the rainfall recharge, recharges from rivers, subsurface horizontal inflow, artificial recharge and inflows from other aquifers (overlying or underlying). Outflows include baseflow to the rivers, outflow of groundwater into the zone of aeration for moisture recovery lost by evapotranspiration, outflow to overlying and underlying aquifers, subsurface horizontal flow, groundwater flow through springs and groundwater pumpage. The response of any aquifer in terms of inflows and outflows can be evaluated through the lumped models (Sokolov and Chapman,1974) or distributed models (Remson et al, 1974; Pinder and Gray, 1977). The lumped aquifer response to known inflows and out- flows can be obtained by designing groundwater storage fluctuations in terms of groundwater head fluctuations and storativity.

The distributed groundwater flow models are based upon the solutions of differential equations governing two-dimensional (Eqn.2.2) or a three-dimensional (Eqn. 2.3a) case like the representing transient groundwater flows in saturated zone. Closed form or series solutions of governing differential equations are available for only idealized boundary and recharge conditions (Bear, 1967). These are generally based upon the assumptions of homogeneity and isotropy. For predicting the aquifer response to the spatially and temporally varying recharge and pumpage under realistic conditions, the governing differential equations have to be solved by appropriate computer-assisted numerical methods. The discretisation of space, necessary for finite difference approximations, can be based upon either any convenient pattern (Tyson and Webor, 1964) or a more stringent rectangular grid pattern. The former is generally known as 'Tyson and Webor' model and the latter as 'finite difference method'.

The finite difference approach introduced by Richardson (1910) involves calculation of approximate solution of partial differential equations. This method has been programmed to solve two-dimensional (Bittenger et al, 1967; Pinder and Bredehoeft, 1968; Pinder, 1970, Prickett and Lonnquist, 1971; Bedinger et al, 1973, Trescott et al, 1976) and three-dimensional (Bredehoeft and Pinder, 1970; Trescott, 1975) transient groundwater flow problems. The implicit form of the finite difference equation generally requires the solutions of large number of linear simultaneous equations and hence the

requirement for storing all the coefficient matrices is enormous. This difficulty is overcome by employing various iterative procedures which primarily bank upon the sparseness of the coefficient matrix.

Some of the more commonly used procedures are alternating direction implicit scheme (Peaceman and Rachford, 1955), Crank- Nicolson scheme (1947), line successive over relaxation (Trescott et al, 1976) and strongly implicit procedure (Stone, 1968). The numerical properties of these methods have been extensively studied (Rushton, 1974; Tomilson and Rushton, 1975; Trescott and Larson, 1977). Murray and Johnson (1977) have demonstrated the second method of linerisation of Boussinesq's equation for simulating the response of unconfined aquifers. The finite element method is reported to overcome many difficulties relating to irregular geometry of the area, heterogeneity and boundary conditions (France, 1974), in addition to giving results of higher degree of accuracy (Javandal and Witherspoon, 1968). Other significant contributions are by Neuman and Witherspoon (1970), Pinder and Frind (1972), Neuman (1973a), Pinder (1973), Pinder and Gray (1977).

2.3.1 Recharge

Recharge rate, an input requirement of all aquifer response models, is one of the important factors controlling the amount of water that may be pumped from an aquifer without depleting it. Rushton (1986) has considered the following equation for describing flow in an aquifer,

$$\frac{\partial}{\partial x}\left[T_x \frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial y}\left[T_y \frac{\partial h}{\partial y}\right] = S\frac{\partial h}{\partial t} - Q \qquad \ldots\ldots (2.5)$$

where h is the groundwater head,

T_x and T_y are the transmissivities,

x and y are the space coordinates,

t is the time coordinate,

S is the storage coefficient or specific yield,

and Q is the recharge intensity

The Richard's equation for estimating groundwater recharge using finite difference method (Krishnamurthi, 1977; Kafri and Asher, 1978) is as follows:

$$\frac{\partial\theta}{\partial t} = -\frac{\partial}{\partial z}\left[K(\theta)\frac{\partial}{\partial z}\left(\frac{-P_c}{\rho g} + z\right)\right] \qquad \ldots\ldots (2.6)$$

where $\frac{-P_c}{\rho g}$ is capillary pressure, θ is moisture content, z and t are spatial and temporal coordinates while $K(\theta)$ is the hydraulic conductivity.

The input data requirement for groundwater modelling would amongst others include transient moisture storage measured as a function of vertical position, permeability-soil moisture curves and evapotranspiration. The approach for the recharge estimation is based upon water budgeting (Eqn.2.7) and field capacity,

$$P = ET + SRF + \Delta SM \qquad \ldots\ldots (2.7)$$

where P is the precipitation, ET is actual evapotranspiration, SRF is the surface run-off and ΔSM is the increase of water availability in root zone. Recharge (Eqn.2.8)is related to current soil moisture storage at field level.

$$\text{Recharge} = \Delta\ SM - DFCT \quad \text{if } DFCT < \Delta\ SM$$
$$= 0 \quad \text{if } DFCT \geq \Delta\ SM \qquad ......(2.8)$$

where DFCT is the current soil moisture deficiency below field capacity.

Some doubts have been expressed regarding the validity of this conventional method (Watson et al, 1976; Rushton and Ward, 1979). Rushton and Ward (1979) have proposed alternative recharge mechanism which allows recharge to occur even when a soil moisture deficit exists. Morel-Seytoux (1976) presented formulae for prediction of pounding time and cumulative infiltration, under influence of rainfall. Reeves (1975) has tested a method of recharge estimation, which assumes that the maximum infiltration rate is simply a function of cumulative rainfall regardless of rainfall versus time history. A few reported studies make use of water table data for the estimation of recharge (Venetis, 1971).

The experimental methods for the measurement of recharge include tritium tracer technique (Vogel et al, 1974; Sukhria et al, 1976) and gamma ray transmission (Singh and Chandra, 1977). An empirical relation between the rainfall and recharge was given by Chaturvedi and Chandra (1961).

2.3.2 Interpolation of Groundwater Head

The need of interpolation arises because the location of the points of groundwater data is very rarely decided on the basis of the requirements of the digital models. Thus, in most of the cases the groundwater heads at the nodal points have to be estimated by interpolating the recorded heads. The conventional graphical and lagrangian methods of interpolation (Ralston, 1965) may not be always suitable because of the subjective bias and artificial undulations associated with the high degree polynomials respectively. Sagar et al (1973, 1975) demonstrated the use of spline functions to approximate the spatial variation of groundwater heads. The closed form functions so obtained can assist in interpolation. Delhomme (1978) has reported the possible use of kriging technique in providing spatial variation of groundwater head. Birtles and Morel (1979) proposed a least square based method of interpolation wherein the unsteady state flow equation is reduced to a steady state by assuming zone-wise spatial uniformity of certain equations.

Kashyap and Chandra (1982) have evolved the least square polynomials of spatial coordinates p and q in any two arbitrarily selected directions to approximate spatial variation of water table data elevations.

2.4 INVERSE PROBLEM

The major hurdle in mathematical modelling of groundwater flow is the collection of adequate data relating to the

spatial and temporal distribution of transmissivity and storativity. Of the difficulties that arise, the principal one is that these properties are not physically measurable quantities (Yeh, 1975). These are, in fact, parameters appearing in the differential equation governing the aquifer response (groundwater head fluctuations) to the aquifer excitation (pumpage, recharge or change in boundary conditions). The estimation of transmissivity and storativity is an inverse process involving the historical excitation, response and associated initial and boundary conditions. The system in this case is represented as in Fig.2.2. A variety of methods exists based on the parameters employed for the estimation of the aquifer response to different forms of excitations with varying conditions and assumptions.

The most widely used method is generally test pumping (Kruseman and Ridder, 1970). This method essentially consists of generating and recording aquifer response under a controlled excitation in the form of pumping from a single well. The generated data are employed to estimate parameters based upon the criterion of 'closest match' between the recorded response and the response given by the governing differential equation. An accurate estimation of the transmissivity and the storativity through conventional pump test methods requires all the real field situations being properly incorporated in the equation involved. For example, Theis (1935) equation assumes axisymmetric radial flow towards a fully penetrating discharging well of infinitesimal diameter in an infinite confined aquifer. Although, several modifi-

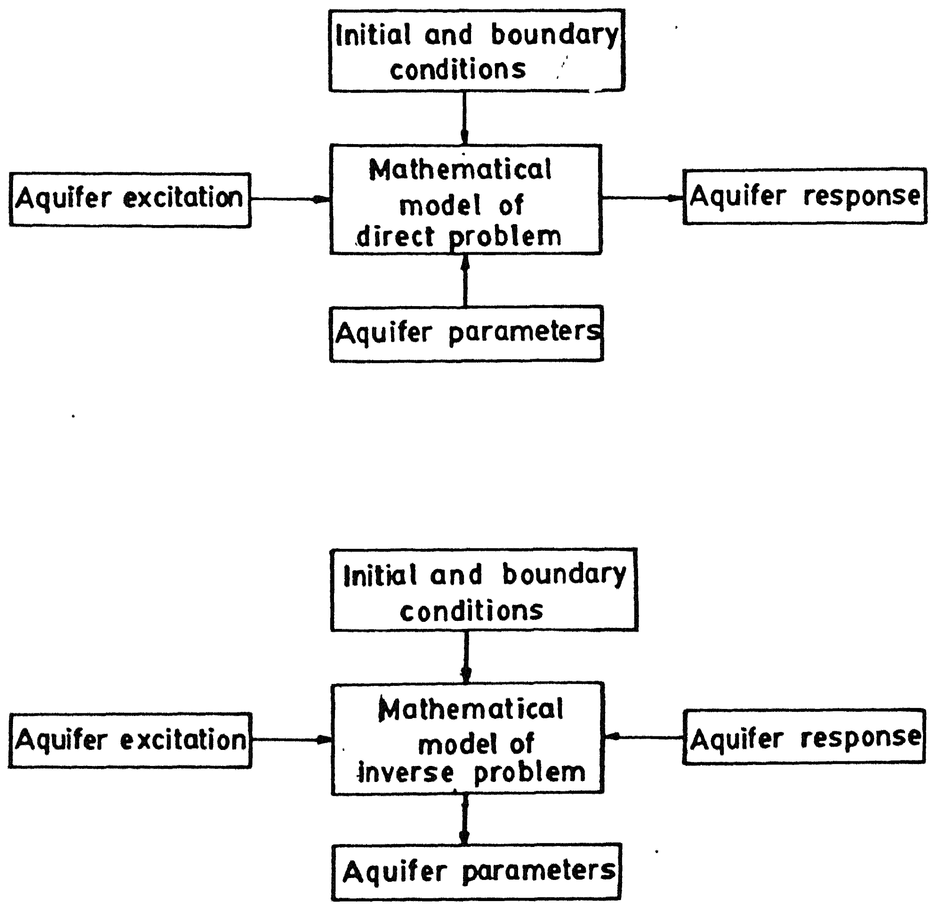


FIG. 2·2 DIRECT AND INVERSE PROBLEMS IN GROUNDWATER HYDROLOGY

cations have been reported (Jacob, 1950; Pinder and Bredehoeft, 1968; Walton, 1970; Yeh and Tauxe, 1971; Marino and Yeh, 1973; Lakshminarayana and Rajagopalan, 1978), there is no single comprehensive method incorporating all the field conditions.

Estimation of transmissivity and storativity can also be carried out using the fluctuations of water levels in hydraulically connected water bodies, along with other form of aquifer exication. These can be viewed as excitations induced by change in boundary conditions. Pinder and others (1969) employed iterative procedures to determine aquifer diffusivity from the historical data relating to the water table elevations and river stage. Yeh (1975) used quasi-linearization (Bellman and Kabala, 1965) to estimate diffusivity of an unconfined aquifer based on historical data of stream-aquifer interaction. Singh and Sagar (1977) developed an analytical method to determine aquifer diffusivity using the Green's function of linearised Boussinesq's equation and specifying an extra boundary condition at the stream-aquifer interface. Sagar and Singh (1978) studied the effect of observational errors in raw data on the values of the aquifer diffusivity, the most restrictive among them being one-dimensional flow, absence of vertical accretion and existence of improper boundary conditions at the other end of the aquifer. These two methods can thus be employed to estimate aquifer parameters of flood plains. The study reported by Venetis (1971) overcomes the assumption of 'no vertical accretion' in a limited way by using uniform

accretion. Other reported contributions are by Ferris (1952) and Rowe (1960).

The estimation of aquifer parameters involves the excitations, response data such as the fluctuations in groundwater heads in space and time in response to known pumping and/or recharge pattern, the initial and boundary conditions. Since the closed form analytical solutions for such complex system are not available, such an approach has essentially to be based on the Boussinesq's equation and is treated as `inverse problem' in groundwater hydrology. There are two types of errors associated with the inverse problem.

1) The system modelling error, as represented by performance criterion,

2) The error associated with parameter uncertainity (Yeh, 1986).

An increase in number of unknown aquifer parameters will generally improve the system modelling error, but will increase the aquifer parameter uncertainity and vice versa. The optimum level of parameterization depends on the quantity and quality of data. The inverse problem is often ill-posed, due to non-uniqueness and instability. Small errors in the head will cause serious errors in the identified aquifer parameter leading to instability.

2.4.1 Classification of Parameter Identification Methods

Various techniques have been developed to solve the inverse problem of parameter identification. Neuman (1973)

classified the techniques into two categories-'direct' and 'indirect'. The direct approach treats the model parameters as dependent variables in formal inverse boundary value problem. The indirect approach is based upon the output error criterion where repetitive numerical solution of Boussinesq's equation with successively modified aquifer parameter values are calculated to reach the closest reproduction of recorded aquifer response under the recorded excitations (withdrawals and recharges) and initial as well as the boundary conditions. The modifications in the aquifer parameter values at the end of each iteration can be based either upon the mathematically rigorous procedures (Knowles et al, 1972) or upon subjective judgment. The latter approach is also known as 'calibration' or 'subjective optimization'.

Kubrusly (1977) classified parameter identification procedures of distributed parameter system into three categories:

1) The direct method, which consists of those methods that use optimization techniques directly to the distributed parameter model

2) Reduction to lumped parameter system, which consists of those methods that reduce the distributed parameter system to a continuous or a discrete time lumped parameter system described by ordinary differential equation or difference equation

3) Reduction to algebraic equation, which consists of those methods that reduce the partial differential equation to an algebraic equation.

2.5 DIRECT METHOD USING THE EQUATION ERROR CRITERION

In the direct methods of solving inverse problem, Boussinesq's equation is directly solved for the aquifer parameters. In practice, observation wells are sparsely distributed in the flow region in an arbitrary fashion and only a limited number of observation wells are available. To formulate the inverse problem by the equation error criterion, missing data have to be estimated by interpolation. The interpolated data along with the observations, which also contain noise, on substitution in the governing differential equation results in an error term (known as the equation error). This error is minimized over the proper choice of parameters (Yeh, 1986).

The equation (Jacob, 1950) for confined aquifer which incorporates anisotropy, heterogeneity and vertical accretion is expressed as

$$\frac{\partial}{\partial x}\left[T_{xx}\frac{\partial h}{\partial x}\right]+\frac{\partial}{\partial y}\left[T_{yy}\frac{\partial h}{\partial y}\right]+Q=S\frac{\partial h}{\partial t} \qquad \text{..... (2.9)}$$

where T_{xx} and T_{yy} are transmissivity values in the direction of principal permeabilities collinear with x and y and Q is the net vertical accretion rate per unit area. The above equation, rewritten with transmissivity as unknown, becomes a first order partial differential equation and can be expressed in the form

$$\frac{\partial h}{\partial x}\frac{\partial T_{xx}}{\partial x} + \frac{\partial h}{\partial y}\frac{\partial T_{yy}}{\partial y} + \left[T_{xx}\frac{\partial^2 h}{\partial x^2} + T_{yy}\frac{\partial^2 h}{\partial y^2} + Q\right] = S\frac{\partial h}{\partial t} \quad \text{.... (2.10)}$$

To estimate transmissivity (T) values from this partial differential equation, values of T at the boundaries designated as Cauchy data (Emsellem and deMarsily, 1971) are needed. This approach can be employed in the estimation of spatially distributed transmissivity values from the groundwater data and vertical accretion data of single time period, provided the first derivatives of groundwater data are elaborate enough. Since a single uncoupled equation can yield the solution of one variable only, this method can not be used to calculate both transmissivity and storativity (S). Either the S or the groundwater data corresponding to steady state conditions has to be an input data. Since in most of the real situations the necessary Cauchy data are not available, this approach has its limitations. This problem can be overcome by employing multiple period data. The overdetermined system, so obtained can be utilized to arrive at optimum values of the parameters, by minimizing the predecided function of the residues in the Boussinesq's equation. This approach introduces the 'smoothening' effects if any discrepancies are present in the raw data.

2.5.1 Solution Procedures

Various methods are available to solve inverse problem by equation error criterion (the direct method) such as energy

dissipation method (Nelson, 1968), linear programming approach (Kleinecke, 1971), the use of a flatness criterion (Emsellem and deMarsily, 1971), the multiple objective decision process (Neuman, 1973), the Galerkin method (Frind and Pinder, 1973), the algebraic approach (Sagar et al, 1975), the inductive method (Nutbrown, 1975), linear programming and quadratic programming (Hafez, 1975), minimization of quadratic objective function with penalty function (Navarro, 1977), and the matrix inversion method allied with kriging (Yeh et al, 1983). To minimize the instability and nonuniqueness, regularity conditions are often required.

Sagar and co-workers (1973,1975) proposed the `algebraic equation method' which dispenses with the requirement of boundary conditions and it is not based on the discretisation of space by nodal points. The spatial derivatives of hydraulic head are estimated by one-dimensional spline function fitted to the observed hydraulic head data. The boundary conditions are eliminated by studying water budget at each individual space point and the parameters estimated by minimizing the sum of squares of residual errors. However this method is useful for the estimation of transmissivity only.

Emsellem and deMarsily (1971) for the first time demonstrated the use of the multiple objectives, in context of aquifer parameter estimation. One of the objectives is to minimize the residue functional and the other is to affect `smoothness' with respect to variation of transmissivity in space. The suggested approach is to significantly reduce the residue functional gradually. The method applies to steady

state conditions and, thus, can yield only the estimates of transmissivity. The idea of multiple objectives has been further extended by Neuman (1973b, 1975), who has proposed a number of criteria to affect 'smoothness' solutions. The problem has been solved by applying linear programming to finite element-based model of groundwater flow.

Nutbrown (1974) has proposed another viable method of estimating transmissivity and storativity. The estimation of transmissivity is done by the water balance of stream-tube for a period of zero storage change. The stream-tube is extended to a turning point (water divide) and the transmissivity at the other end is determined. The storativity is obtained by employing the data of the period by displaying the storage changes. This approach was further extended by Britles and Morel (1979) who incorporated 'equivalent steady state' concept for interpolation. Other significant contributions are the study of input of additional data on calibration by Gates and Kisiel (1974), regression-based drawdown model by Maddock III (1976) and the subjective information by Lovel and co-workers (1972). Aquifer excitation along with transmissivity and storativity was estimated by Tison (1965), Monech and Kisiel (1970), Venetis (1971), and Sagar et al (1975).

2.6 INDIRECT METHOD USING THE OUTPUT ERROR CRITERION

The criterion used in this approach is generally the minimization of a 'norm' of the difference between observed

and calculated heads at specified observation points (Yeh, 1986). It has both advantages and disadvantages. While it can be applied with limited data and does not require differentiation of the measured data, the disadvantage of this approach is that the minimization is usually nonlinear and non-convex. The algorithm starts from a set of initial estimates of the parameters which improves with each iteration until the system model response is sufficiently close to the observed data.

2.6.1 Solution Procedures

Gauss-Newton procedure for inverse problem solution was first applied by Jacquard and Jain (1965), followed by Jahns (1966). Along with quadratic interpolation, this method was used by Thomas et al (1972) and also adopted by Yoon and Yeh (1976a, 1976b), Yeh and Yoon (1981) and Sadeghipour and Yeh (1984) along with Rosen's gradient projection. Conjugate gradient oriented method proposed by Gauss-Newton and further modified by Marquardt has been used by Gavalas et al (1976). Cooley (1977) modified Gauss-Newton method using nonlinear regression by linearisation. Cooley (1982) applied this method with the help of prior information to solve inverse problem. Shah et al, (1978) and Sun and Yeh (1985) have also used Gauss-Newton method to estimate aquifer parameters.

Steepest descent method was first adopted for inverse problem by Vemuri and Karplus (1969). Chen et al (1974) used the same method along with conjugate gradient. Chavent and

co-workers (1975) have applied this method without conjugate gradient for oil reservoir. Conjugate gradient was alone applied to solve inverse problem by Neuman (1980).

Quasi linearisation was first used by Yeh and Tauxe (1971) to identify aquifer parameters. Later Marino and Yeh (1983) and Distefeno and Rath (1975) have applied the same method to solve inverse problem. The other methods of solving inverse problem worth mentioning are Newton-Raphson method used by Neuman and Yacowitz (1979), maximum likelihood and kriging method of solution by Kitanidis and Vomvoris (1983), and cokriging by Hoeksema and Kitanidis (1984), Gaussian conditional mean and kriging as compared by Hoeksema and Kitanidis (1985). Quasi linearisation, maximum principle gradient, influence coefficient and linear programming algorithms were compared by Yeh (1975a). Yeh (1975b) used quadratic programming to solve inverse problem.

It can be proved that least square solution approach yields the most likelihood solutions when the errors in raw data are normally distributed (Beck and Arnold, 1977). This least square form of the residue functional, necessitates the use of nonlinear programming. Cooley (1977) proposed the method of regression for the parameters of two-dimensional steady state flow domain and also analyzed the goodness of fit and significance of computed parameters in a subsequent paper (Cooley, 1979). Kleinecke (1971) adopted 'mini-max' and the 'sum of the moduli' functionals and used the linear

programming to obtain optimal values of the aquifer parameters. The main drawback of this method is that the final solutions may not include the parameters of all the grid points. For example, out of 144 nodes, 44 did not generate an estimate of storativity. Results of transmissivity are even more sparse, since out of the 676 estimates, 56 percent were not in the solution domain. Subsequently, Navarro (1977) modified the Kleinecke's method to make use of field information relating to the values of transmissivity and storativity.

Haimes and others (1968) employed decomposition and multilevel optimization, to arrive at optimal parameters. This method assumes an infinite aquifer. Labadie (1975) proposed the term 'surrogate parameters' meaning that the estimated parameters may be quite different from the aquifer properties, since those parameters represent a sort of lumped effect of aquifer properties and many other unaccounted local features like heterogeneity and complex boundary conditions.

Kashyap and Chandra (1982) have developed a numerical scheme to estimate quantitatively the parameters related to geohydrological and hydrological characteristics of aquifers, employing historic data like hydraulic head, rainfall and pumpage. This scheme is based upon constrained minimization of the sum of squares of residuals in the Boussinesq's equation. Derivatives of the hydraulic head are estimated by polynomial approximation of least squares.

Thus, the inverse problem for the estimation of aquifer parameters from aquifer excitation and other related initial and boundary conditions can be solved by using equation error criterion (direct method) or output error criterion (indirect method). In the present work, the direct method has been adopted for the estimation of aquifer parameters. As already indicated, groundwater head values at various nodal points as obtained by interpolation from available field data are needed as inputs. The procedure for the interpolation and smoothening of water table head data is outlined in the next chapter.

CHAPTER 3

WATER TABLE HEAD RECONSTRUCTION

3.1 INTRODUCTION

For quantitative evaluation of groundwater potential, historic record of water level data is a prerequisite. The available records usually consist of groundwater head measured periodically in observation wells (open wells or piezometers). These discrete point data can be employed to generate continuous functions which approximate the true functional relationships between the groundwater head and the spatial coordinates in the domain bounded by the problem area. Such approximate functional relationships can be greatly useful in subsequent analysis of the data, to estimate aquifer parameters. A deterministic method for generating such approximation has been developed in this chapter. The method consists of regression of groundwater head data by minimizing sum of squares of residual errors, with relevant statistical tests.

3.2 PROCESSING OF WATER TABLE DATA

The observation well data can be processed to yield a variety of information relating to the groundwater movement,

storages and groundwater head elevations at various locations in the problem domain. There are three basic numerical techniques for this data processing viz., differentiation, integration and interpolation.

The differentiation of groundwater head data with respect to space is necessary because of the linear velocity-hydraulic gradient relation incorporated in Darcy's law. Thus, the first derivatives of groundwater head with respect to space i.e. the hydraulic gradient needs to be calculated for the estimation of subsurface velocities and recharge across the boundary. Similarly, the second spatial derivatives of groundwater head assist in calculation of the imbalance between horizontal inflows and outflows at any space point across the boundary.

The integration of groundwater head in space is required to be carried out for estimating the water released from or taken into the storage in a given area for a certain period. The interpolation becomes necessary, when the nodal points of a distributed parameter groundwater flow model do not coincide with the observation points. The available data of the observation points are employed to assign groundwater heads at the nodal points.

3.3 EXISTING METHODS

The mathematical operations mentioned above can be performed by arriving at a spatial variation of groundwater head using stochastic, deterministic or graphical method. Kriging, a stochastic method (Delhomme, 1978) assumes that the

groundwater system is too complex to be explained by analytical expressions. For deterministic system analysis, the spatial variation can be arrived at by graphical procedures or functional approximation. Alternatively, finite differences can be employed to perform the interpolation, differentiation or integration.

The graphical procedures essentially involve the drawing of contours of equal groundwater heads by visual interpolation of the groundwater head elevations at the observation points. This procedure, though employed quite frequently, has two major handicaps in that a strong subjective bias and a subconscious assumption of linear variation of groundwater head between two observation points are involved. These handicaps become all the more pronounced when the data points are sparse or non-uniformly distributed in space. The finite difference methods of differentiation, integration and interpolation are procedures which, in most of the cases, afford the estimations of error bounds. However, the requirement of data along the two orthogonal directions is rarely satisfied.

The 'functional approximation' approach consists of approximating the true functional relation between the groundwater head and space coordinates by an explicit function which is directly amenable to the mathematical operations. Functional approximation can be either 'exact type' i.e. no residues at the observation points or the 'least-square' type (Ralston, 1965) with residues at the observation points. These procedures require the total number of coefficients in the

approximating function to be equal to or less than the total number of data points. Where the general form of the functional relation is not known, a polynomial may be used to approximate the function in a domain, provided it is known a priori that the function is continuous. This approach is validated by the Wierstrass's theorem (Ralston, 1965). The lagrangian methods of `exact' functional approximation in one-dimension by a polynomial can be suitably extended to a two-dimensional situation. This method, however, requires a polynomial of very high degree which apart from increasing the computational efforts involved in the numerical operation of the function, can also cause large artificial undulations of the functional estimate in between the observation points.

The spline functional approximation (Sagar et al,1973, 1975) involves passing piece-wise continuous polynomial function through known functional values at the observation points with the compatiability conditions at the interfaces of the adjacent polynomials satisfied. This approach is one of numerical curve-fitting methods. It overcomes the necessity of employing higher degree of polynomial but may still cause artificial undulations if the observation well data are not completely error-free, as they would rarely be. The differentiation of these approximating functions to arrive at the derivatives of the true function, may involve large errors originating from even small amplitude of `noise' present in the raw groundwater head data.

The magnitudes of noise are not known but the frequency distribution will be near Gaussian if the sample size is large

enough and there are no systematic errors (Beck and Arnold, 1977). In such situations, the smoothening of `noise' in the raw data can be accomplished by adopting a `least square polynomial' rather than an exact polynomial, provided the adopted polynomial closely approximates the true functional relation between the groundwater head and space coordinates in the given domain of space. The relevance of smoothening processes can be directly inferred from the smooth water table or piezometric surfaces encountered in physical situations of alluvial aquifers (Neuman, 1973; Sagar et al, 1975).

Kriging, a spatial interpolation scheme, was used to obtain the head variations in the flow domain (Yeh, Yoon and Lee, 1983) as a presampling filter to reconstruct the head distributions over the spatial computational grids (or nodes). Kriging, when used with direct approach (Sagar, 1975) for parameter identification, replaces repetitive numerical simulations of the governing equations in the optimization approach.

Kitanidis and Vomvoris (1983) applied Kriging to provide minimum variance and unbiased point estimates. The statistical approach developed by them has the significant advantage in that it does not attempt to restore details which can not be actually extracted from available data.

3.4 PROCEDURE ADOPTED

The locations for the available groundwater data generally would not coincide with all the grid points. So it

is necessary to interpolate the missing groundwater heads at the nodal points of the computational grid with the help of least square approximation followed by statistical tests. The coordinates (x,y) of the observation points and head values at those points constitute the input data. Before going for interpolation it is mandatory to normalize observation points.

3.4.1 Least Square Polynomial Approximation

The spatial and temporal variation of groundwater head in an unconfined aquifer is governed by an equation of the diffusion type. Therefore, one may consider the observed head values available at finite number of points in space and time to be made up of two components i.e.

$$h^{*}_{i,k} = h_k\,(p_i,q_i) + E \qquad \begin{matrix} i= 1,2,\ldots n \\ k= 1,2,\ldots m \end{matrix} \qquad \ldots\ldots\ (3.1)$$

where $h^{*}_{i,k}$ = observed head at i^{th} observation point (p_i,q_i) and at k^{th} time,

$h_k\,(p_i,q_i)$ = the true value of head at (p_i,q_i) and at k^{th} discrete time,

(p,q) = function of space coordinates,

p,q = solution of groundwater flow equation at k^{th} discrete time,

E = random error associated with observations and assumed to be statistically un-correlated,

n = number of observation points,

and m = number of time periods

For real situations, it may not be possible to know the functions of $h_k(p,q)$ explicitly. However, it is known a priori

that these functions are continuous in space. The continuity of these functions can be directly inferred from the continuity of water table or piezometric surfaces which do not show any spatial discontinuity in aquifers free of hydrogeological structures. Thus, the function $h_k(p,q)$ can be approximated by polynomials of spatial coordinates p and q, and by $H_k(p,q)$ (least square polynomial solution). By choosing an appropriate degree of the approximating polynomial, the difference between the true function and the approximating one can be restricted to a small but non-zero value i.e.

$$\left[\left| h_k\ (p,q) - H_k\ (p,q) \right| \right] \leq \delta_e \qquad \ldots\ldots (3.2)$$

$k = 1$ to m and $\delta_e > 0$, where δ_e is the error

For obtaining a least square solution, the total number of the coefficients in a chosen polynomial must be less than the total number of data points. The coefficients of the approximating polynomials are estimated employing the observed groundwater head data ($h^*_{i,k}$), by the least square criterion which implies the estimation of the coefficients in the chosen equation such that the sum the squares of the deviations of the observed values from the numerically predicted ones is minimized i.e.

$$\text{minimize} \sum_i \left[h^*_{i,k} - H_k\ (p_i,q_i) \right]^2 \qquad \ldots\ldots (3.3)$$

The residues at the i^{th} observation point at k^{th} time point are given by

$$e_i = h^*_{i,k} - H_k(p_i, q_i) \qquad \ldots\ldots (3.4)$$

A complete estimation of errors (E) and one to one representation of $h_k(p,q)$ by $H_k(p,q)$ can be ensured only if the residues (e_i) are identical to the errors in the raw data (E). However, such a situation does not generally occur because of the following reasons :

(1) The true form of the function $h_k(p,q)$ is not known and it is being approximated by $H_k(p,q)$. According to the Wierstrass theorem, the minimum difference between the two functions within the domain of the observation, will always be grater than zero.

(2) Even if the true form of function $h_k(p,q)$ were unknown, (e_i) will be identical to (E) only if the latter are normally distributed. Because of the finite sample size, the frequency distribution of the errors may be approximately normally distributed.

3.4.2 Degree Of Polynomial

As the degree of polynomial is increased, it may get closer and closer to the true functional relation between the groundwater head and space coordinates in a given domain. However, beyond a certain limit, any further increase in the degree of polynomial may only induce the approximating least square polynomial to conform to the 'noise' pattern of the raw data instead of providing a better approximation of the true groundwater head. A partial polynomial may provide a better approximation than a full polynomial. It is desirable to

arrive at an optimal form of the polynomial i.e. a partial polynomial of minimum possible degree and terms. Thus, apart from providing a better approximation, it also minimizes the computational efforts. The optimal form of the approximating function can be arrived at by employing the statistical tests of significance (Daniel and Wood, 1971). The standard tests of significance used in the context of multiple regression, can be directly applicable to least square polynomials provided the 'independent variables' (i.e. the terms containing the different exponents of the spatial coordinates) are not correlated to each other (Dooge, 1973). This is applicable if the spatial coordinates and hence the independent variables are known exactly (without any error) and are thus, deterministic in nature.

The degree of polynomial can be decided on the basis of criterion on minimum standard error s.

$$s^2 = \sum_i e_i^2 / (n - np) \quad \text{where } n > np \qquad \ldots\ldots (3.5)$$

where

e_i = residue at i^{th} observation point (Eqn 3.4),

$\sum e_i^2$ = sum of the squares of the residues (SSR),

n = total number of the data points,

$(n - np)$ = the degree of freedom (DF) of the proposed least square polynomial,

and np = number of coefficients in the polynomial.

As the degree of the polynomial is increased, there will be a reduction in SSR but at the same time DF will also

decrease since the third degree and a fourth degree full polynomials would have ten and fifteen terms respectively. The polynomial which gives minimum standard error (s) is tentatively chosen as the approximating function (Ralston, 1965).

3.4.3 Truncation Of Polynomial

For any given level of confidence, the confidence limit for each of the estimated coefficients can be defined. This criterion can be used to delineate the terms which contribute significantly towards the explanation of the power of polynomial model.

The confidence limits of estimate for coefficient of the i^{th} term are given by :

$$b_i \pm t\,(\,n - np\,,\,\alpha/2\,)\; s.e\,(b_i) \qquad \ldots\ldots\ldots \quad (3.6)$$

where $t(\,n-np,\,\alpha/2)$ is $(1-\alpha)$ percentile of t-distribution with $(n-np)$ degree of freedom.

The standard error of b_i, s.e. (b_i) is given by

$$s.e\,(b_i) = s\,\sqrt{c_{ii}} \qquad \ldots\ldots\ldots \quad (3.7)$$

where c_{ii} is the i^{th} diagonal element of the corrected sum of squares and product matrix.

The statistics for testing $(b_i=0)$ is

$$t = \frac{b_i}{s.e\,(b_i)} \qquad \ldots\ldots\ldots \quad (3.8)$$

Null hypothesis is accepted if the computed value of `t' is lower than the critical value of `t' for the degree of freedom (n-np) at some chosen level of confidence (generally 95 percent). Acceptance of null hypothesis implies that the i^{th} term does not have significant explaining power and may be dropped. Thus, a few terms of polynomial can be dropped by adopting the students `t' test. The test is described by

$$t = \frac{\overline{X} - \mu_0}{s / \sqrt{n}} \qquad \ldots\ldots (3.9)$$

where $\overline{x}$ = mean of the sample,

μ_0 = hypothetical mean of the population (18 percent),

n = number of observations,

and s = standard deviations of the observations.

By this test, a reduced model can be prepared. However, it has to be tested if the reduced model (RM) gives as good of fit as full model (FM). Coefficients of the polynomial with reduced number of terms and SSR are reestimated. If K_d is the number of terms dropped then, `F' statistic is estimated by

$$F = \frac{[\,SSR\ (RM) - SSR\ (FM)\,]\ /K_d}{SSR\ (FM)/\ (n - np\ )} \qquad \ldots\ldots (3.10)$$

If the calculated F is large in comparison to thetabulated values of F with K_d and (n-np) degrees of freedom at α percentile level of confidence, the result is significant at level α i.e. the RM is unsatisfactory. It implies then, even though each of the terms dropped individually does not have

significant explaining power, collectively these terms explain a significant part of the variation in the groundwater head.

3.4.4 Acceptability of the Approximation

Subsequent to the estimation of coefficients, it is desirable to make an assessment of the goodness of fit. The most widely used index is the multiple correlation coefficient R, defined by

$$R^2 = 1 - \frac{SSR}{\sum_{i=1}^{n} (h^*_{i,k} - \bar{h}_k)} \qquad \ldots\ldots\ldots (3.11)$$

where $$\bar{h}_k = \sum_{i=1}^{n} \frac{h^*_{i,k}}{n} \qquad \ldots\ldots (3.12)$$

R^2 represents the fraction of the initial variance explained by the model. The adequacy of the proposed fit to approximate the true function may be checked by examining the residues. The standard residue at i^{th} station is given by

$$es_i = e_i / s \qquad \ldots\ldots\ldots\ldots (3.13)$$

The standard residues have zero mean and unit standard deviation. In general when the model is correct, the standard residues fall between +2 and -2 and their plots against independent variables i.e. x and y do not exhibit any trend. Even after satisfying this criterion for the standard residues, some anomalous values still exist at some space points due to the presence of hydraulic discontinuities in the aquifer. They originate from geological structures like

faults, folds and dykes. As a result of such structures, the water table will show marked differences in its elevations. Such groundwater heads can not be adequately approximated by least square polynomials. The discontinuities as evidenced from the contour map can be related to causative features identified from geological maps or remote sensed data. In these situations, estimation of derivatives can be monitored on either side of the structure by creating a temporary boundary between two space points in the problem domain.

If the least square polynomial provides a good fit to the observed data, the hydraulic continuity of the aquifer is established. The 'goodness' of least square fit is characterized by high correlation coefficient (low standard errors), random distribution of standard residues in space and absence of outliers. The orientation and distribution of outliers can give some idea about the location and delineation of discontinuities, if present. However, the occurrence of outliers without any systematic orientation in space can be due to some localized phenomena like incomplete recuperation at the time of observation and the inadequate hydraulic connection between the well and the aquifer.

3.5 PROCESSING OF THE WATER TABLE DATA IN THE STUDY AREA

The water table data used in this study are collected from Andhra Pradesh State Groundwater Department, Hyderabad. This data pertaining to a period of ten years (June78 to May88) was at fifteen day intervals. There are 69 observation wells in the study area (Fig.3.1 and Table 3.1). The data is

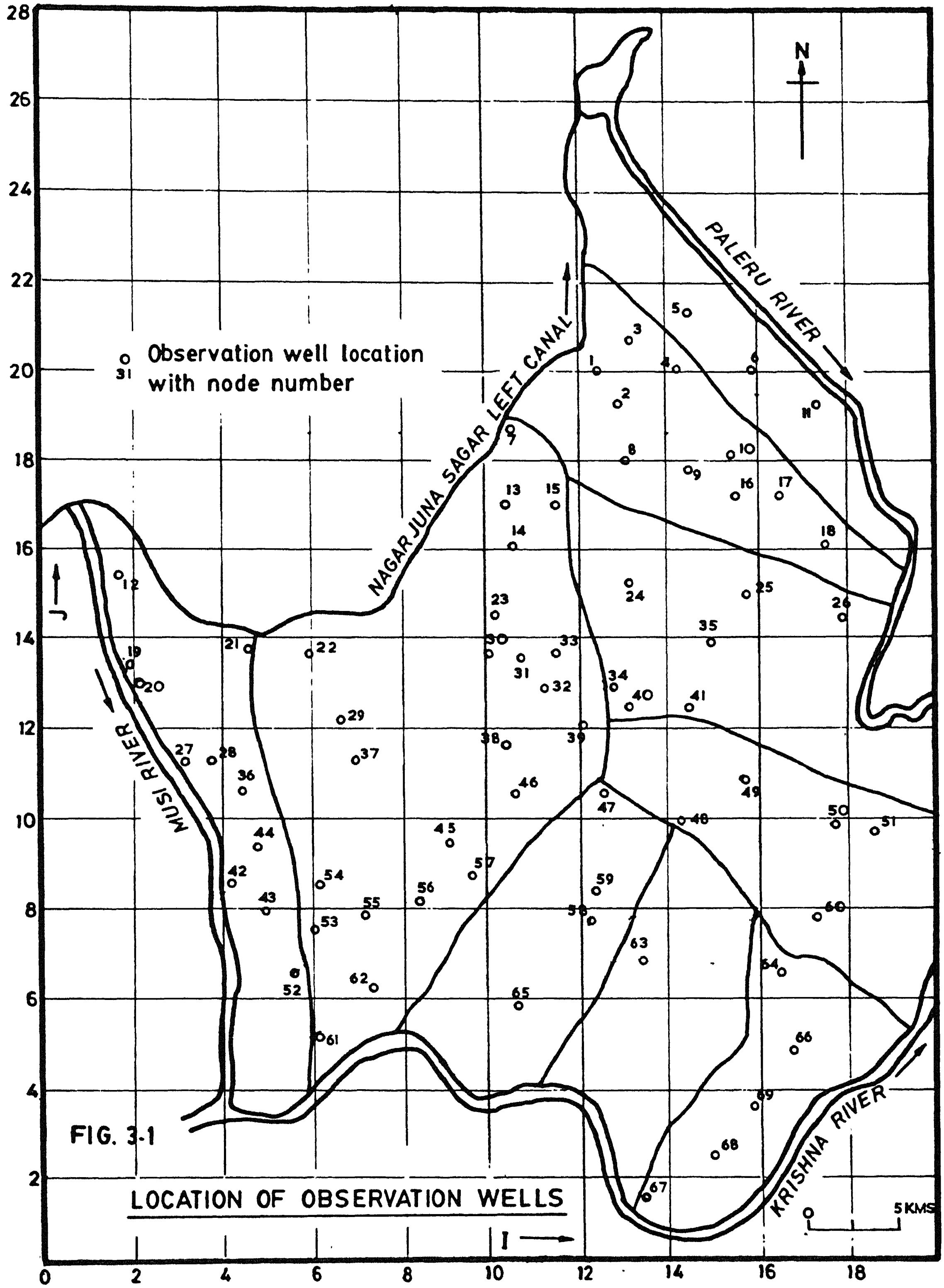


FIG. 3-1

LOCATION OF OBSERVATION WELLS

Table 3.1 : Details of Observation Wells Chosen for Data in the Present Study

Sl.No.	location	depth below ground level (m)	formation(s) tapping
1.	Yekalaskhana peta	8.05	granite
2.	Thalleballi	7.35	granite
3.	Ratnapuram	11.00	granite
4.	Vasanthapuram	8.30	granite
5.	Tripuraram	8.00	granite
6	Anatagiri	9.60	granite
7.	Barakathgudem	7.75	granite
8.	Akupamula	9.40	granite
9.	Komarabanda	8.20	granite
10.	Kahanpur	8.20	granite
11.	Shantinagar	7.15	granite
12.	Dacharam	7.60	granite
13.	Lakkavaram	9.20	granite
14.	Bethavolu	15.00	granite
15.	Mukundapuram	8.10	granite
16.	Kodad	7.30	granite
17.	Tammabandlapalem	7.80	granite
18.	Dorakuntla	6.80	granite
19.	Somavaram	8.75	granite
20.	Somavaram Tandla	7.65	granite
21.	Fathepur	6.60	granite
22.	Ponugodu	10.90	granite
23.	Thallamalkapuram	4.65	granite
24.	Chilkooru	6.25	granite
25.	Kothagudibanda	10.00	granite
26.	Kapugallu	8.50	granite
27.	Chillepalli	6.52	granite
28.	Neereddicherla	6.45	granite
29.	Gardepalli	9.20	granite
30.	Keethavarigudem	10.00	granite
31.	Rayanagudem	10.40	granite
32.	Gopalapuram	9.60	granite
33.	Boorugugadda	8.60	granite
34.	Huzurnagar	9.80	granite
35.	Togarai	6.70	granite
36.	Dirsencherla	8.30	granite
37.	Garikuntla Tanda	7.20	granite
38.	Yelapalasingaram	9.65	granite
39.	Lingagiri	10.35	granite
40.	Yeriyaram	6.35	granite

.... Contd....

Table.3.1 : Contd...

Sl.No.	location	depth below ground level (m)	formation(s) tapping
41.	Ganapavaram	9.50	granite
42.	Yellapuram	8.60	quartz arenite
43.	Palakeedu	8.25	granite
44.	Gudiguntlapalem	8.60	granite
45.	Amararam	10.75	granite
46.	Srinivasapuram	8.20	granite
47.	Anandnagar	8.61	granite
48.	Medlacheruvu	11.90	granite
49.	Kandibanda	9.85	quartz arenite
50.	Revoor	6.55	limestone
51.	Dondapadu	10.90	limestone
52.	Komatikuntla	10.45	quartz arenite
53.	Bothapalem	7.80	granite
54.	Kamalacheruvu	6.00	granite
55.	Alangapuram	6.60	granite
56.	Yethavakella	8.50	granite
57.	Allipuram	6.95	granite
58.	Mattampalli	9.00	quartz arenite
59.	Choutapalli	13.65	granite
60.	Gudimalkapuram	16.90	limestone
61.	Zonapadu	14.70	limestone
62.	Gundlapadu	7.20	quartz arenite
63.	Raghunadhapalem	9.50	limestone
64.	Mallareddigudem	22.15	limestone with shale
65.	Peddavedu	11.00	quartz arenite
66.	Shobhanadrigudem	14.10	limestone with shale
67.	Chitryala	20.20	limestone
68.	Nakkagudem	13.85	limestone with shale
69.	Tammvaram	12.95	limestone with shale

Well locations are indicated in Fig.3.1 with the same serial numbers.

divided into two slabs of five years each. The data from both the five year sets (1978 to 1983 and 1983 to 1988) were used to estimate aquifer parameters by inverse problem. These estimated parameter values in turn have been utilized in calculating the water table elevations for the same periods. The estimated water table elevations were compared with the observed values to ascertain the accuracy of the model. The procedure involved in water table data processing is described in the succeeding pages.

Least square polynomials were generated to approximate the spatial variation of water table elevation in sixty sequential months from June 1978 to May 1983. By this approximation a coefficient matrix was calculated. This coefficient matrix was employed to estimate spatial and temporal derivatives of hydraulic head. The axes (I,J) of the coordinate system along with the origin are indicated in Fig.3.1.

Procedure for the estimation of the minimum standard error to determine the degree of polynomial has been described (Section 3.4.2). On the basis of this error as a first attempt, coefficients of the third degree were estimated. A full polynomial of third degree would have ten terms for each period, as expressed below.

$$H_k(p,q) = b_1p^3 + b_2q^3 + b_3p^2q + b_4p\,q^2 + b_5p^2 + b_6q^2 + b_7p\,q + b_8p + b_9q + b_{10} \quad \text{.... (3.14)}$$

Using the above polynomial, the summary statistics were calculated for all the sixty months. The space points which displayed a standard residue of ±2 are treated as outliers. The `t' values were computed using the equation 3.8. These computed values of `t' are compared with the critical values of `t' at 95 percent confidence level and at degree of the freedom equalling the number of data points retained minus the number of terms. It was found that values of computed `t' of almost all the terms other than the constant term were lower ~~than~~ than the corresponding critical `t' values. Depending upon this criteria, some of the space points were deleted and the coefficients were once agian calculated. This procedure helped in increasing the value of the variance but the standard error is too large to be acceptable. Moreover, large standard error indicates presence of large residues. As a next step, the polynomial terms showing large residues are deleted together with the space points and the coefficients are once again estimated. The output indicated the standard error to be within the range of ±2 without any outliers. Even the space points showed improvement with this partial polynomial. But to achieve the satisfactory values of above criterion, many space points were deleted and polynomial equation had to be truncated.

To include all the values of space points and to have a better approximation, it was necessary to go for a higher degree of polynomial. A fourth degree polynomial having fifteen terms was considered. The full polynomial of fourth degree is of the form

$$H_k(p,q) = b_1p^4 + b_2q^4 + b_3p^3q + b_4p\,q^3 + b_5p^2q^2 + b_6p^3 + b_7q^3$$
$$b_8p^2q + b_9p\,q^2 + b_{10}p^2 + b_{11}q^2 + b_{12}p\,q +$$
$$b_{13}p + b_{14}q + b_{15} \qquad \ldots\ldots\ldots \quad (3.15)$$

The coefficients of polynomials are calculated, using the same criterion as in the third degree case. This scheme is also tested for decreasing the number of outliers and minimizing the standard error. In this case too, some space points are deleted and some terms of polynomial are not considered after correlating their 't' values with the critical 't' values at 95 percent confidence level. This truncated polynomial exhibited better spatial distribution and had lesser number of outliers than full polynomial. When it is tested for F test, the results are encouraging and revealed that the dropping of some space points and/or the polynomial terms leads to an improved approximation of spatial variation of water table data.

As an example, the data for the period June 1978 to May 1979 can be examined. These data at 69 observation wells were tested for the above procedure. The least square polynomials for the twelve months were calculated in the beginning with a third degree full polynomial with all the ten terms and at all the 69 space points. These coefficients are tested for summary statistics and the results are shown in Tables 3.2 and 3.3. The computed 't' statistics for the month of February'79 are shown below as a typical example.

Table 3.2 : Deletion Scheme of Space Points for a Third degree Polynomial

Sl.No.	month	number of points deleted	variance initial	variance final	goodness of fit initial	goodness of fit final
1.	Jun 78	16	0.9102	0.9742	81.55	95.33
2.	Jul 78	16	0.9256	0.9820	81.32	95.28
3.	Aug 78	17	0.9009	0.9725	81.54	95.46
4.	Sep 78	16	0.8690	0.9537	82.24	95.48
5.	Oct 78	17	0.9033	0.9621	81.18	95.35
6.	Nov 78	17	0.9033	0.9680	82.84	95.42
7.	Dec 78	15	0.8704	0.9593	82.14	95.42
8.	Jan 79	15	0.8743	0.9602	82.27	95.83
9.	Feb 79	15	0.8910	0.9714	82.04	95.68
10.	Mar 79	15	0.8975	0.9657	82.05	95.55
11.	Apr 79	15	0.8975	0.9648	82.08	95.57
12.	May 79	15	0.9067	0.9733	81.93	95.23

The common space points deleted are of serial numbers 4,8,6,11,13,15,18,25, 26,27,35,38,39,55 and 67 on Fig.3.1.

Table 3.3 : Truncation of Polynomial Terms for a Third Degree Polynomial

Sl.No.	month	number of terms deleted	variance		standard error(m)	
			initial	final	initial	final
1.	Jun 78	3	0.9102	0.9742	0.9032	0.7791
2.	Jul 78	3	0.9256	0.9820	0.9222	0.7216
3.	Aug 78	2	0.9009	0.9725	0.8937	0.6328
4.	Sep 78	2	0.8690	0.9537	0.9072	0.6219
5.	Oct 78	3	0.9033	0.9621	0.8873	0.6855
6.	Nov 78	2	0.9033	0.9680	0.9050	0.7234
7.	Dec 78	1	0.8704	0.9593	0.8593	0.5866
8.	Jan 79	2	0.8743	0.9602	0.9769	0.6838
9.	Feb 79	3	0.8910	0.9714	0.9012	0.7243
10.	Mar 79	3	0.8975	0.9657	0.9312	0.7465
11.	Apr 79	2	0.8975	0.9648	0.9060	0.6791
12.	May 79	2	0.9067	0.9733	0.8955	0.6954

The common terms deleted are p^2q, pq^2 and pq.

Term	t value
p^3	-0.94
q^3	-1.20
p^2q	0.25
pq^2	-0.34
p^2	1.25
q^2	-1.17
pq	-0.45
p	0.90
q	-2.36
constant	30.50

The standard error is 0.9012.

The variance value is about 0.8910 and the goodness of fit is estimated as 82.038. As the space points 4, 8, 6, 11, 13, 15, 18, 25, 26, 27, 35, 39, 55 and 67 have displayed deviation and presence of outliers, they were deleted and the coefficents were computed again. The statistics of fit for the rest of the points is as follows:

Term	t value
p^3	1.22
q^3	2.56
p^2q	0.48
pq^2	-0.34
p^2	1.53
q^2	-2.23
pq	0.89
p	-1.54
q	-3.45
constant	564.23

As per the criterion, the ploynomial terms showing residuals of ±1 i.e. p^2q, pq^2 and pq are deleted. After this truncation, the coefficients and the summary statistics were once again calculated. While the standard error of partial polynomial fit worked out to be of a low value (0.7243 m) the variance value rose to 0.9714. The goodness of fit has also increased to 95.677. But to attain these results, 15 space points and three terms had to be curtailed. It may be noticed that in each month's data, large number of residuls (about 15 points) are present (Table 3.2). Thus the inadequacey of third degree polynomial in approximating the spatial variation of water table during these months is established. This neccessiated the recomputation of the least square polynomials for all the months using the fourth degree polynomial. This polynomial was truncated on the lines as described earlier. The space points truncated are 4, 13, 35 and 67. The final fourth degree partial polynomial has got 13 terms and 65 space ponits, after deleting the terms p^3q and p^2 depending on the criterion adopted for the removal of residuals.

This partial fourth degree poynomial equation was used to approximate the spatial variation of water table head values at all 69 points in the study area for the 120 months period. The head values are then interpolated using the equation

$$h_k\ (p_i, q_i) \simeq H_k\ (p, q) \qquad \text{..... (3.16)}$$

The head values thus processed are stored for later use to assist in the calculation of the second spatial derivatives and the first temporal derivatives. With help of these

derivatives, the inflows and outflows across the boundary and the changes in storage as well as the aquifer parameters are estimated, the processes for which are described in Chapters 4 and 5.

The computer program to estimate the least square polynomials and to calculate groundwater system elements (Chapter 3) has been developed by the author in the course of present study. The algorithm for the same is presented in Appendix A.

3.6 CONTOURING AND TREND SURFACE PLOTTING

Contour Maps

Construction of contour maps of spatially distributed data presents problems similar to those in the creation of equally spaced data from the observations located at irregular intervals along a line, since the first step in contouring usually is to produce a regular mesh or grid of control points (Davis,1973). The regular grid may be produced in a variety of ways, ranging form estimates at grid points based on nearest observation to estimates derived from trend surfaces of all observations.

A set of values to be contoured is entered into the computer as (3 X n) array, where each data entry corresponds to X_1 coordinate, X_2 coordinate, and Y or dependant variable which will be contoured. Thus, the inputs to the contour program are given in terms of an east-west coordinate (X_1), a north-south coordinate (X_2) and elevation of water table above

datum plane (Y) for each of the observation points which are expressed by numerical coordinate system. Then, a rectangular grid of points is created which will control the computing process. This grid will become a rectangular mesh of equally spaced points estimated from the data. The grid is determined by the equation

$$\hat{Y}_k = \frac{\sum_{i=1}^{n} (Y_i/D_{ik})}{\sum_{i=1}^{n} (1/D_{ik})} \qquad \ldots\ldots\ (3.17)$$

where $\hat{Y}$ is estimated elevation at k^{th} grid point, n is the number of observation points and D_{ik} is the distance from observation point i to grid point k.

$$D_{ik} = \sqrt{(X_{1k} - Y_{1i})^2 + (X_{2k}-X_{2i})^2} \qquad \ldots\ (3.18)$$

where X_{1k} and X_{2k} are grid points corresponding to X_{1i} and X_{2i} observation points.

In ordinary contouring routine, after establishing grid points and corresponding values the next step would be to interpolate between adjacent grid points and determine the coordinates of any contour line that may pass between two points and this is done by linear interpolation. A simple algorithm for printing contour maps on a line printer can be adopted or with the help of a graphical package, contours can be drawn using a dot-matrix printer.

The contour map is helpful in finding out water table position at each grid point. Pronounced changes in the hydraulic continuity as evidenced on such a map can be attributed to causative geological features, if present, or to the erroneous interpolation during least square polynomial approximation.

Trend Surface Plotting

A trend surface is a surface generated by a linear function of the geographical coordinates of a set of observations, so constructed that the squared deviations from the trend are minimized (Davis, 1973). The trend is a linear function. That is, it has a form $\hat{Y} = bX_1 + bX_2 + \ldots.$, where the b's are coefficients and the X's are some combination of the geographic coordinates. The equation would yield values of the $\hat{Y}$ which are trend components of an observation such as the groundwater head. The specific linear function chosen for trend must minimise the squared deviations from the trend. A linear trend surface is represented by an equation of the type

$$Y = b_o + b_1X_1 + b_2X_2 \qquad \ldots\ldots. (3.19)$$

A geologic observation, Y, may be regarded as a linear function of some constant value (b_o) which is defined as

$$b_o = f\ (b_1, b_2) + n \qquad \ldots\ldots. (3.20)$$

where b_1 and b_2 are east-west and north-south components and n is the number of observations. Since three unknowns are involved in this equation, three simultaneous equations are needed to solve the same.

$$\left.\begin{aligned} \Sigma\, Y &= b_o^n + b_1\Sigma\, X_1 + b_2\Sigma\, X_2 \\ \Sigma\, X_1Y &= b_o\,\Sigma\, X_1 + b_1\Sigma\, X_1^2 + b_2\Sigma\, X_1X_2 \\ \Sigma\, X_2Y &= b_o\Sigma\, X_2 + b_1\Sigma\, X_1X_2 + b_2\Sigma\, X_2^2 \end{aligned}\right\} \quad \ldots\ldots \quad (3.21)$$

The above equations can be rewritten in the matrix form as

$$\begin{bmatrix} n & \Sigma\, X_1 & \Sigma X_2 \\ \Sigma\, X_1 & \Sigma\, X_1^2 & \Sigma X_1X_2 \\ \Sigma\, X_2 & \Sigma\, X_1X_2 & \Sigma X_2^2 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} \Sigma Y \\ \Sigma X_1 Y \\ \Sigma X_2 Y \end{bmatrix} \quad \ldots\ldots \quad (3.22)$$

A least square line may be expanded to a second degree curve (parabola) by adding a squared term to the linear equation.

$$Y = b_0 + b_1X_1 + b_2\, X_2^2 \quad \ldots\ldots\ldots \quad (3.23)$$

An equivalent expression of (Eqn.3.23) gives a second-degree trend surface

$$Y = b_0 + b_1X_1 + b_2X_2 + b_3X_1^2 + b_4X_2^2 + b_5X_1X_2 \quad \ldots\ldots \quad (3.24)$$

This procedure can be extended for higher degree of trend surfaces, with more unknown coefficients.

A trend surface program consists of three parts:

(1) A routine to generate the matrix of sums and cross products.

(2) A simultaneous equation solver or a matrix inverter.

and (3) A plot program.

Grid points (X_1, X_2 values), dimensions of the map and scaling factor (D_{ik}) can be calculated as in contouring

program. But the observation values (Y) are employed, after fitting them to a second or higher degree of trend surface equation, to generate a trend surface map. The goodness of fit of a trend surface may be tested statistically, by comparing the variance due to regression or trend to the variance due to deviations from the trend. The contour plots are prepared with the help of a graphical package (PLOT 88) in MSFORTRAN and the trend surface diagrams are generated on IBM PC/AT with the help of a EGA driver on the basis of the program written in C language as developed by the author.

3.7 CONFIGURATION OF THE WATER TABLE IN THE STUDY AREA

Water table contour maps for the months of April, August and December of 1981 are presented (Figs.3.2, 3.3 and 3.4.) to indicate typical pattern for one year in the study area. While there is a variation of the head values in the three cases at different points, the patterns are in general similar in all the three cases. Several distinct trends can be noticed. A general gradient of the water table head from north to south occurs with variation from over 150m (above MSL) to 56m (above MSL) in the southeastern corner of the area.

Effect of geology on the water table configuration is evidenced on overlapping the water table map on a geological map. For example in the area where dykes are present, a modification of the slope is pronounced as in the areas south of Ponugodu, east of Kamalacheruvu, north west of Lingagiri and south west of Ponugodu (Fig.3.2). A marked change in the head values in the vicinity of the dykes is noticeable in the

granitic terrain within these areas.

Effect of lineaments on the water table head configuration has also been evidenced. In the granitic terrain west of Bethavolu, marked change in the water table contour plots can be seen in the region where the lineament is located (Fig.3.2). Similarly within the limestone terrain a NE-SW lineament in the area north of Chitryala appeares to have influnced the contours in widening their spacing indicative of increased permeability.

Typical trend surface plots for water table head variation at 4 years intervals are indicated in Figs.3.5, 3.6 and 3.7. It is observed that in all the three cases, the trends are similar.As seen in the contour maps, although a change in the head values exist at different periods, while the present vertical scale of the trend surface plots the differences are not pronouncedly brought out in the figures.To provide an idea of topography in the area, the mesh for topography has also been incorporated (white grids in Fig.3.5).

Thus, in the present chapter reconstruction of water table head data has been carried out by minimizing sum of the squares of the residual errors. A functional approximation was considered with necessary statistical tests to estimate the least square polynomials. The data were processed and the coefficients were calculated. These data were later used to estimate the groundwater system elements (Chapter 4) and aquifer parameters (Chapter 5).

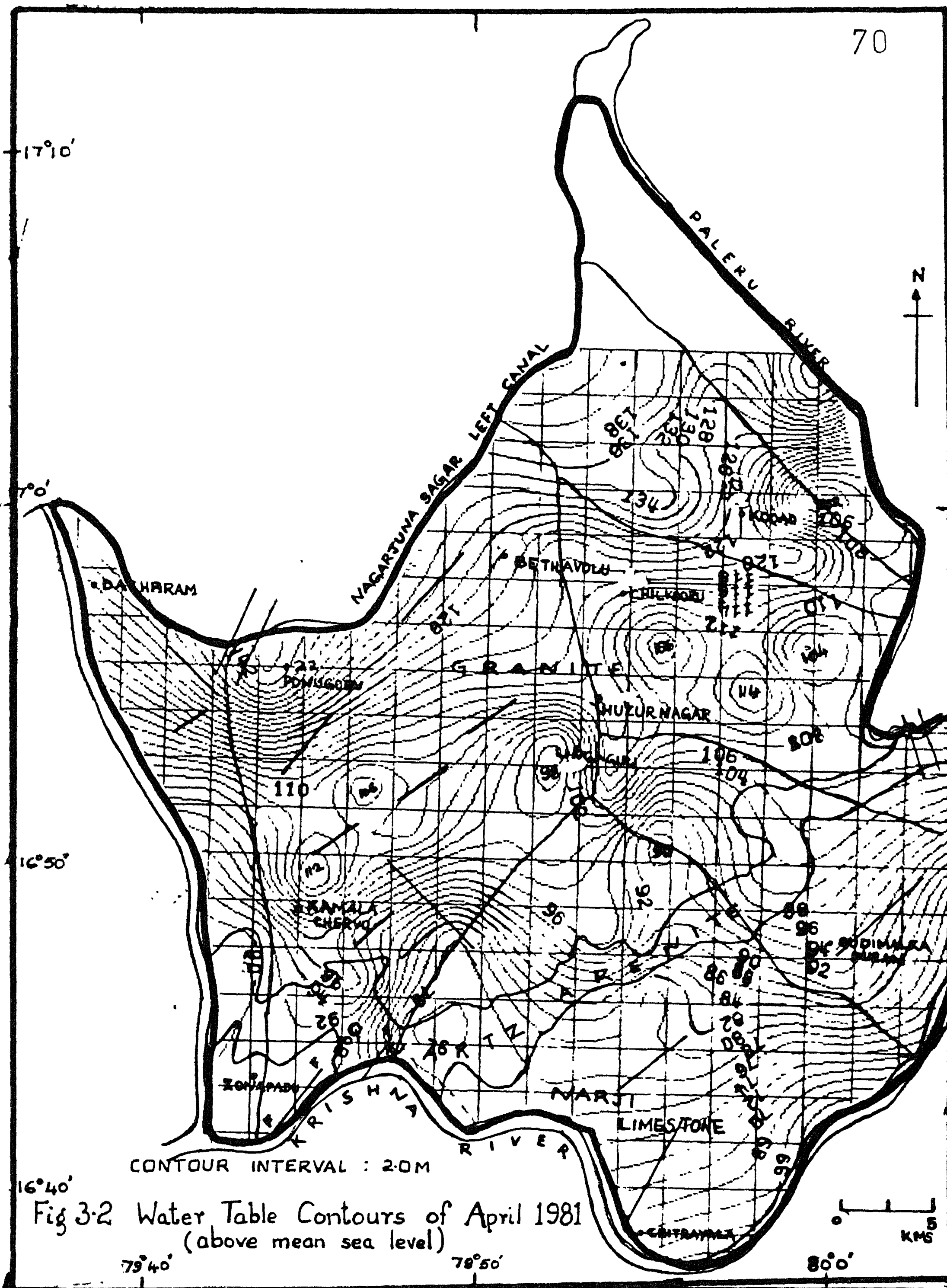


Fig 3·2 Water Table Contours of April 1981 (above mean sea level)

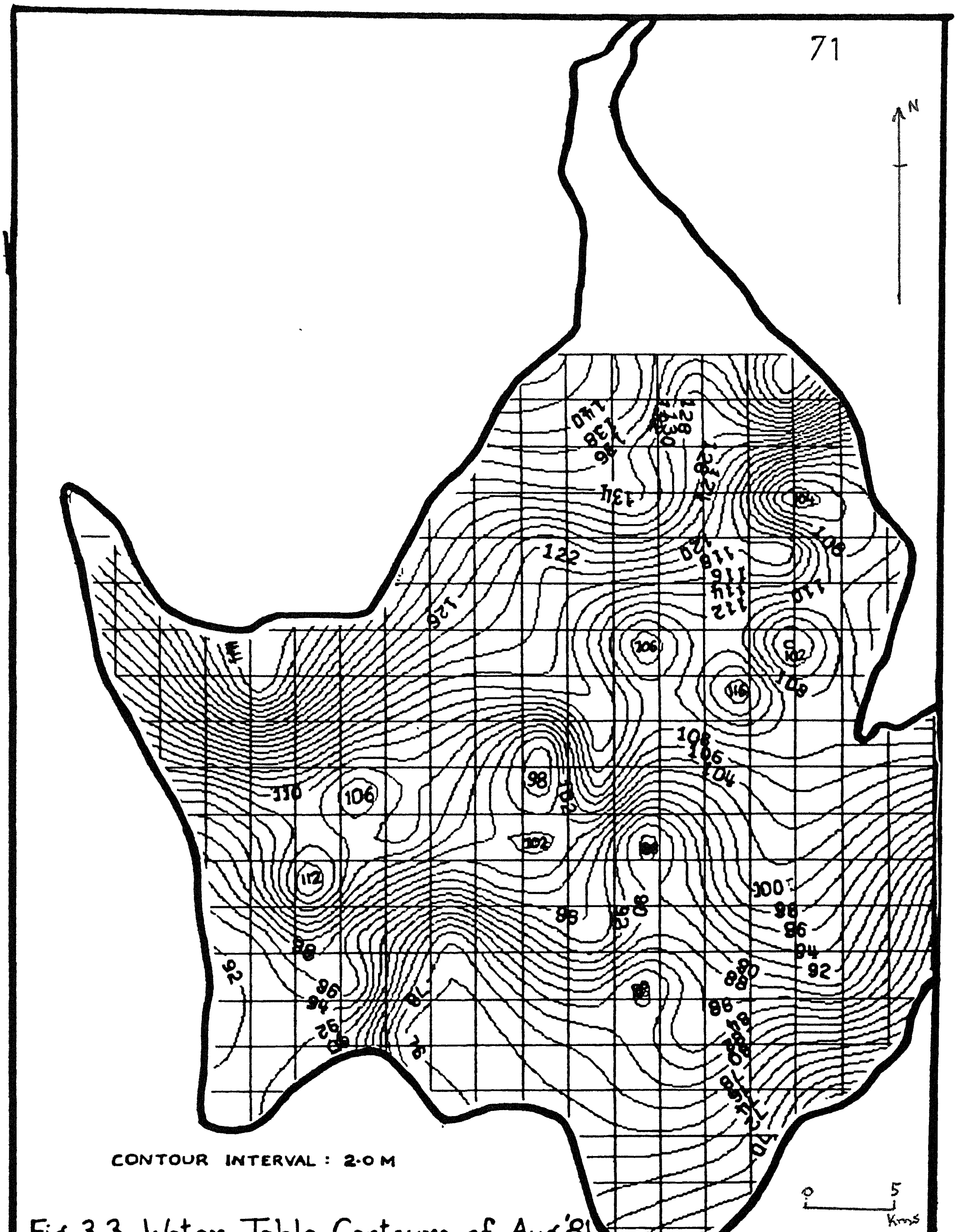


Fig 3·3 Water Table Contours of Aug '81 (above mean sea level)

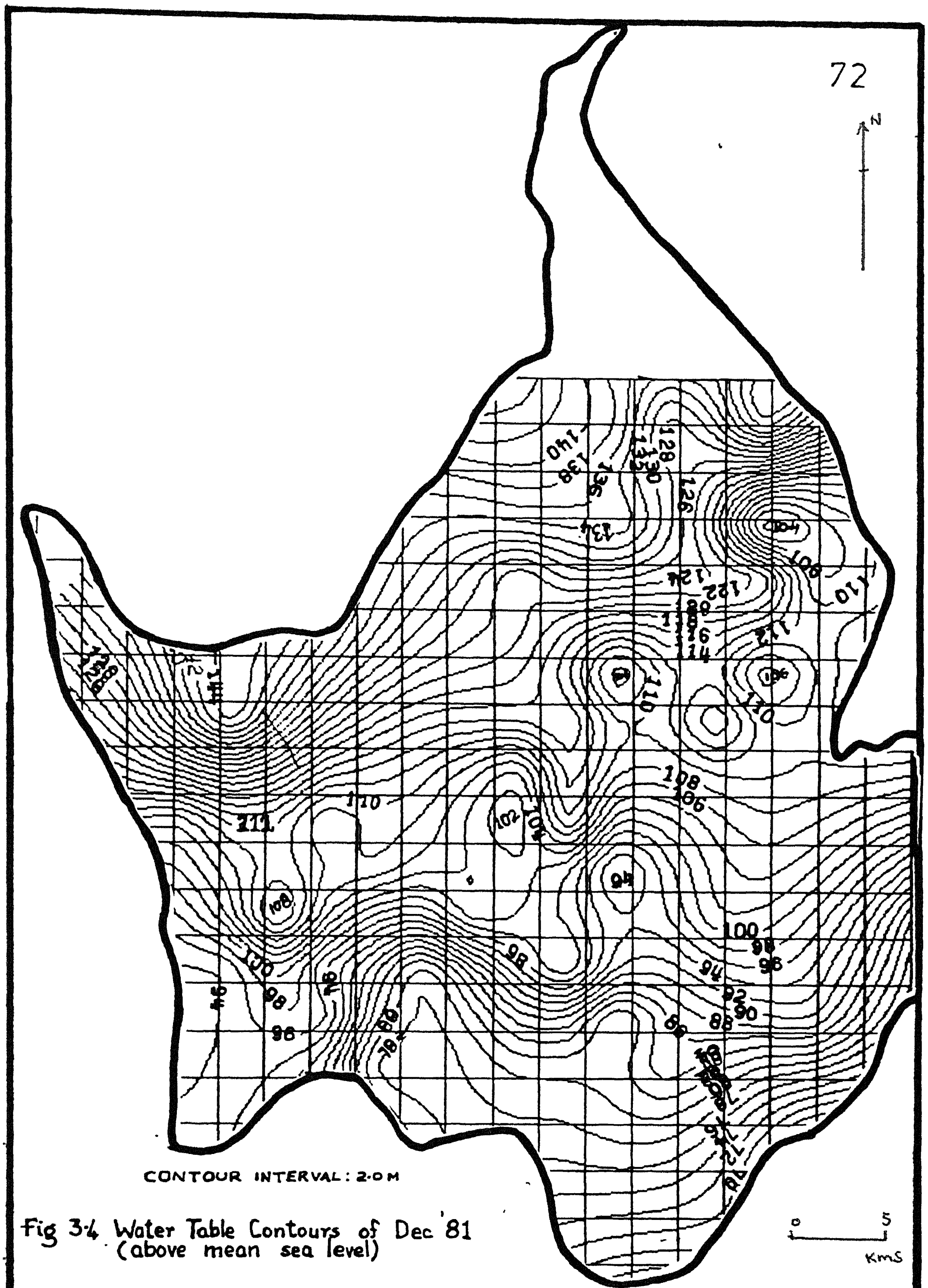
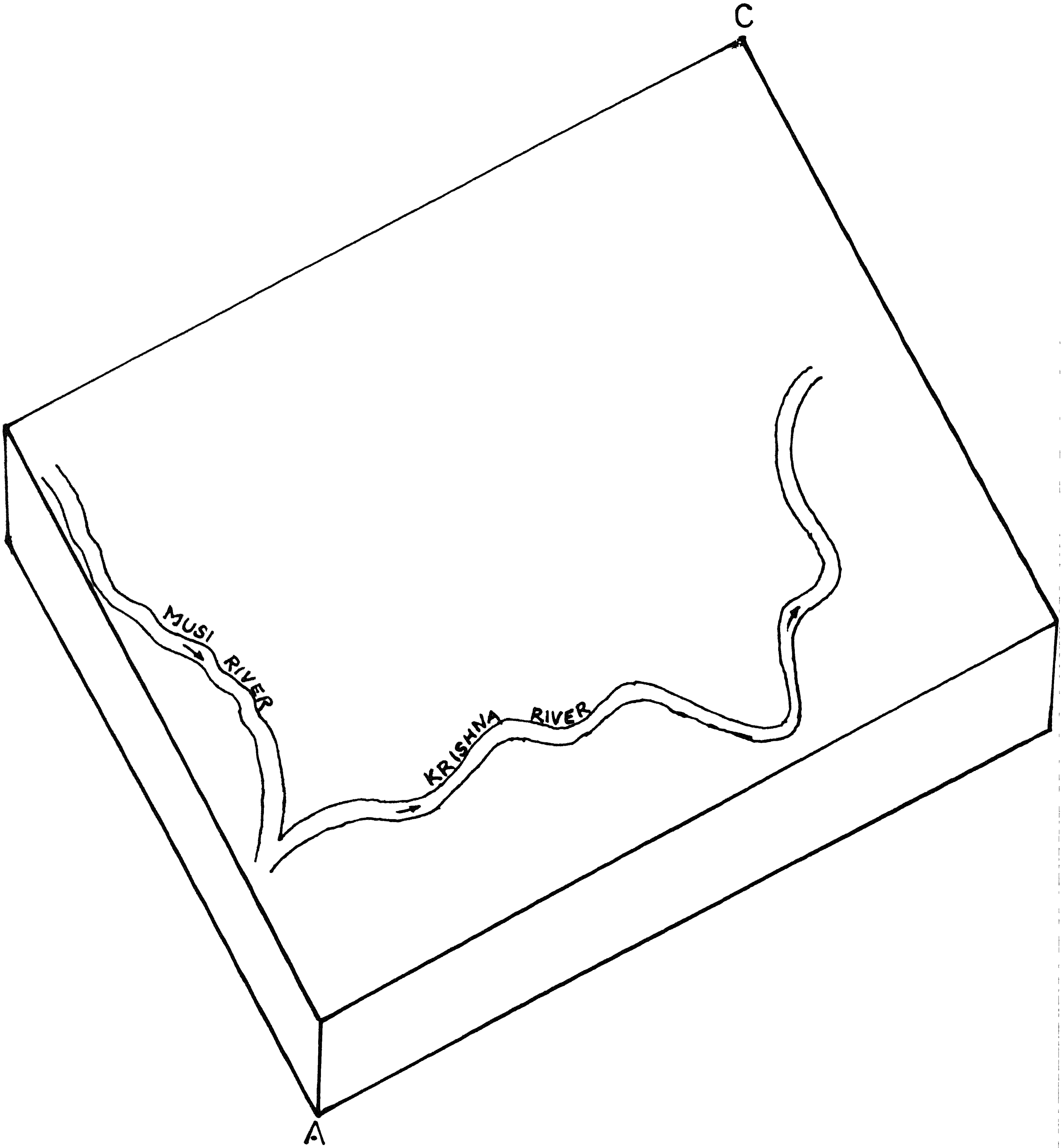
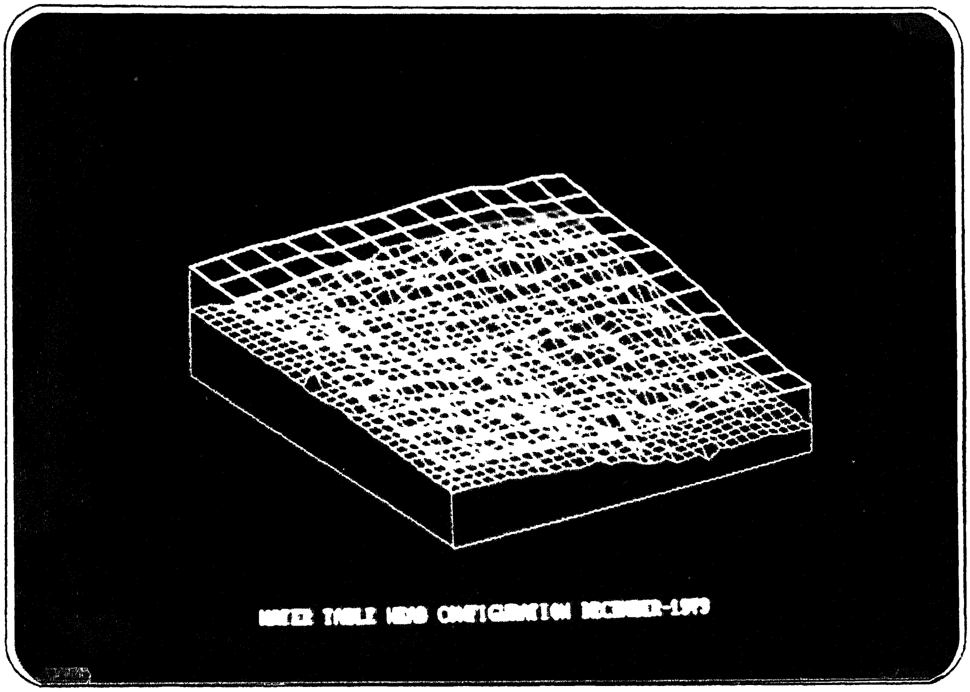


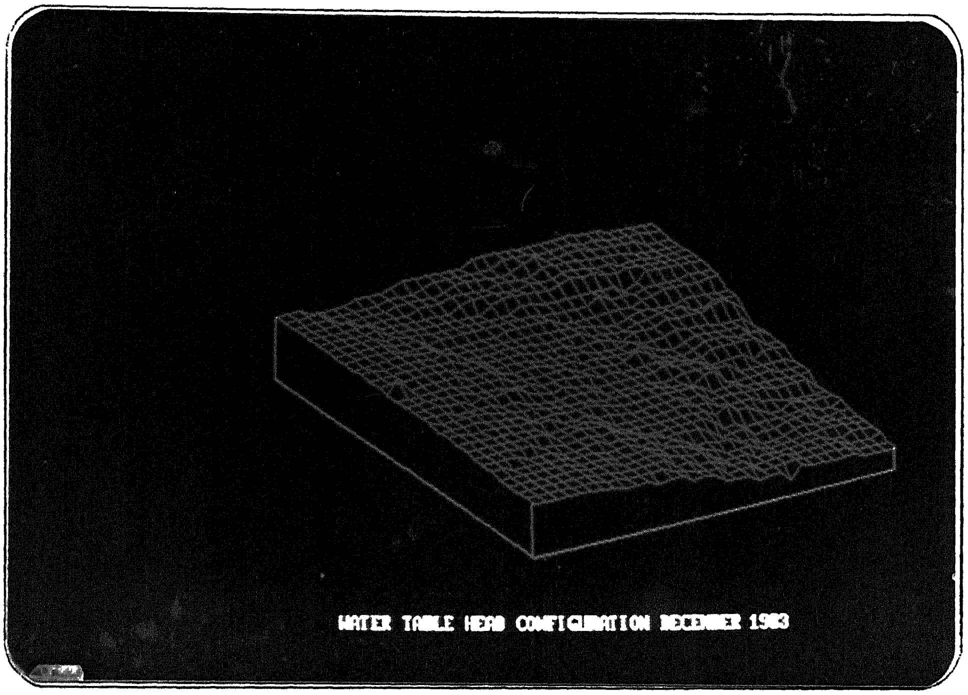
Fig 3.4 Water Table Contours of Dec '81
(above mean sea level)



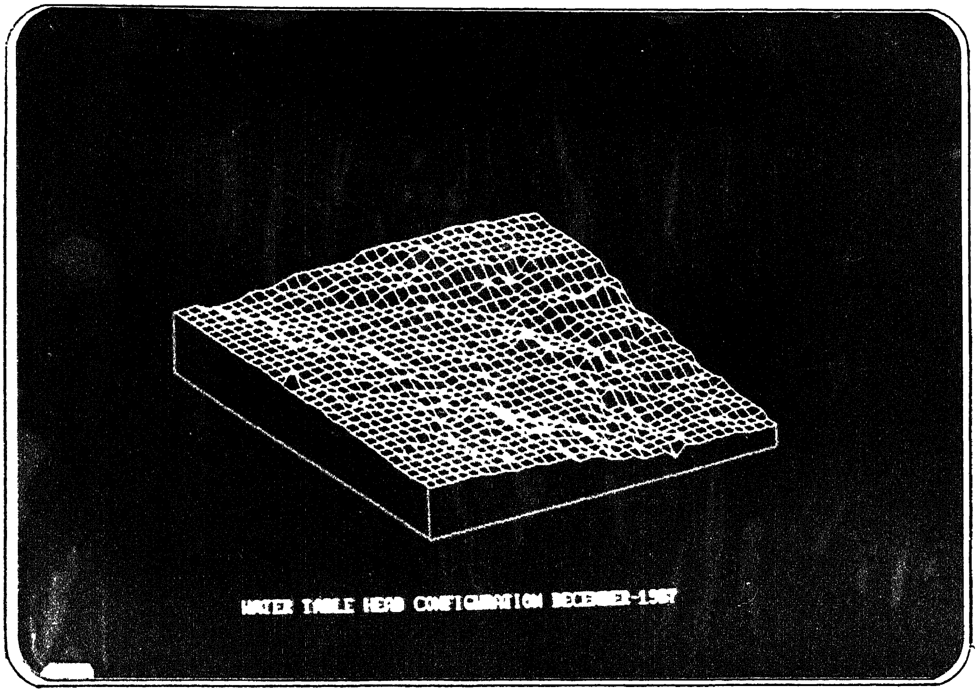
ORIENTATION OF TREND SURFACE



3.5 Water Table Head Configuration December, 1979



3.6 Water Table Head Configuration December, 1983



3.7 Water Table Head Configuration December, 1987

CHAPTER 4

ESTIMATION OF GROUNDWATER RECHARGE AND DISCHARGE COMPONENTS

4.1 INTRODUCTION

For proper evaluation of aquifer response to any given excitation, the various recharge and discharge elements are to be estimated. These elements are from the surface as well as groundwater source. Rainfall, canal seepage, evaporation and irrigation effect are from surface source while the groundwater system elements include flow across the boundary, storage and well pumpage. The algebraic sum of these values form an important input in addition to the water table elevation data in groundwater modelling. The groundwater head values obtained by processing (Chapter 3) are used for the estimation of flow across the boundaries and storage. In this chapter, procedures for the estimation of all the system elements are presented and the same are estimated for the study area.

4.2 ELEMENTS OF GROUNDWATER BALANCE

The hydrologic equation based on the law of conservation of matter, as applied to hydrologic cycle defines the total

water balance. Groundwater balance deals with aspects of balancing various components of groundwater supply (recharge) and disposal (discharge) with storage changes in the ground water reservoir. The items of recharge and discharge are as follows:

Components of recharge

1. Precipitation, infiltrating to the aquifer
2. Natural recharge from streams, lakes and ponds
3. Groundwater inflow
4. Artificial recharge from canals and irrigation

Components of discharge

1. Evaporation from the shallow aquifers and transpiration from plants.
2. Natural discharge by effluent seepage, spring flow and base flow to streams and lakes.
3. Groundwater outflow .
4. Artificial discharge by pumping or through drains.

The groundwater balance of a basin or an area, for an inventory period may be expressed as

Groundwater inflow - groundwater outflow = change in groundwater storage.

4.2.1 Estimation of Recharge Components

The methods adopted for estimation of recharge take into account intensity and duration of rainfall, evapotranspiration, soil moisture, runoff, infiltration capacities of

soils, storage characteristics of aquifers and water level fluctuations and movement of ground water. The various recharge components to be estimated and the methods employed are discussed in the following paragraphs.

Infiltration occurring at the land surface can be estimated by the soil moisture balance approach. The soil-moisture balance for any time interval can be expressed as

$$P = AE + I + R + \Delta S_m \qquad \ldots\ldots (4.1)$$

where

P = rainfall,

AE = actual evapotranspiration,

ΔS_m = change in soil moisture storage,

I = infiltration,

and R = surface runoff

Soil-moisture budgeting, taking into account evapotranspirational abstraction from precipitation, provides a measure of moisture available for runoff and infiltration. This can be done by Thornthwaite's formula. In this method measurements of field capacity and wilting point are to be measured from the available moisture down to the root zone.

For long duration of rainfall and under conditions of excess of infiltration capacity, the hydrologic equation can be expressed in the following form provided, the losses due to evapotranspiration are taken neglegible.

$$P = R + W_p \qquad \text{......... (4.2)}$$

where

P = rainfall,

R = surface runoff,

and W_p = recharge by infiltration from rainfall.

The rainfall infiltration factor can be determined for catchments with varying terrain, soil and other characteristics. The following equation can be used to determine the rainfall-infiltration factor

$$I_p = (R_{bf} + GW_e + E_t + E + W_{as} - W_{ag} - S_p) \qquad \text{...... (4.3)}$$

where

I_p = rainfall infiltration factor,

R_{bf} = base flow,

GW_e = groundwater extraction,

E_t = evapotranspiration from groundwater,

E = evaporation from base flow,

W_{as} = recharge due to infiltration of applied surface water,

W_{ag} = recharge by return circulation from applied groundwater,

and S_p = stream-bed percolation.

Water-level hydrographs can be decomposed to obtain fluctuations in response to rainfall and other inputs of water to the aquifer. The rise of water level in an aquifer represents the net response to a process of simultaneous drainage or discharge from and recharge to the aquifer. In order to find the actual recharge, the water-level recession has to be extended till the period of maximum rise of water level. If the recharge is discontinuous with large time gaps, the water level will decline between two recharge periods. The total recharge will be the sum of individual rises. The water table fluctuation observed in response to rainfall in a single year may not be representative of average long term fluctuations and recharge. The average rate of groundwater recharge to water table aquifer can be estimated from the shape of water table of an aquifer bounded on two sides by parallel streams (canals) of infinite length. If the aquifer is homogeneous and isotropic and is recharged at a constant rate of accretion in space and time, the rate of recharge (W) can be written in the form

$$W = \frac{T}{2.73 \times 10^{-5}\left[\frac{ax}{h_o} - \frac{x}{2h_o}\right]^2} \qquad \ldots\ldots (4.4)$$

where

W = rate of recharge (cm/year),

a = distance (m) from stream to the groundwater divide,

x = distance (m) from the stream to an observation well,

h_0 = elevation (m) of the water table at the observation well with respect to mean stream level,

and T = transmissivity (m^2/day).

4.2.2 Recharge from Canals and Streams

Recharge through percolation from canals and streams depends on the infiltration capacity of the canal and stream bed and sides as well as subsurface lithology, extent of wetted perimeter, length of canal or stream, discharge, sediment load, physical and chemical properties of water, and relative position of the bed with respect to the water table. As in the case of streams, canals may be influent or effluent and it is essential to establish the relation before and during the studies. Recharge rates may decline over the years due to water-logging or clogging of pores of the bed material.

For quantitative estimation of percolation rates, a stretch of canal or stream with no diversion of water is generally selected. In case of canals, the seepage losses are expressed usually in m^3/s per million sq m of wetted surface as

$$I = \frac{Q_1 - Q_2}{A} \qquad \text{..... (4.5)}$$

where

I = seepage losses in cumecs per million sq.m,

Q_1 = discharge at upstream point, cumecs,

Q_2 = discharge at downstream point, cumecs,

and A = area of wetted surface (sq.km), (length of canal X

average of perimeters between two discharge points).

Recharge from canals and streams that are in direct hydraulic connection with a phreatic aquifer underlain by a horizontal impermeable layer at shallow depth can be determined by Darcy's equation, provided the flow satisfies Dupuit assumptions.

$$Q = K \frac{h_s - h_1}{L} A \qquad \ldots\ldots (4.6)$$

Where h_s and h_1 are saturated thickness values of the aquifer over a distance L at the stream or canal. For calculating the area of flow cross-section, the average of the saturated thickness $(h_s + h_1)$ is usuaslly taken.

4.2.3 Estimation of Discharge Components

Once the total groundwater recharge has been estimated, the groundwater losses have to be accounted for a quantitative estimation of the exploitable surplus.

For the estimation of subsurface outflow of groundwater, contour maps of groundwater surface prepared on the basis of the water table level data of wells located both within and outside the section delimiting the basin outlet are used.

The quantity of groundwater discharge appearing as base flow can be measured by stream gauging at the basin outlet. Evapotranspiration from fully saturated soils with vegetation equals the potential evapotranspiration rate. Evaporation loss

from soil surface forms one of the major components of groundwater losses and is pronounced in the water-logged canal command areas. Draft from individual wells may vary widely depending on the yield, type of well, well design, depth of water level, method of lift, crops grown etc.

4.3 RECHARGE CALCULATION

In general, the rainfall recharge and groundwater pumpage are the dominant factors influenceing the recharge estimation. However, in canal command areas seepage from canals contributes significantly to the net recharge along with other components of Q (total algebraic sum of inflows and outflows). Thus, for Q in i^{th} space and K^{th} time period can be written as

$$Q_{ik} = R_{ik} - W_{ik} + C_{ik} + X_{ik} \qquad \ldots\ldots (4.7)$$

where R_{ik} is the rainfall recharge, W_{ik} is the effective withdrawl due to pumpage (with adjustment to the return flows), C_{ik} is recharge from canals through seepage and X_{ik} is algebraic sum of all other X_{ik} recharge components of Q_{ik}, with the recharge taken as positive and discharge taken as negative values respectively.

In tropical regions, the recharge from rainfall is an important component of groundwater balance. The available rainfall records are usually adequate to estimate the spatial and temporal distribution of rainfall.

In the present study, the following equation proposed by Kashyap (1981) was used to calculate recharge elements of the aquifers.

$$R_{ik} = f_r \left[P_{i,k}, P_{i,k-1}, \ldots\ldots\ldots, P_{i,k-me}, \alpha^i \right] \quad \ldots\ldots \quad (4.8)$$

where $P_{i,k}$ is the rainfall at i^{th} space point during k^{th} period, me is the number of periods preceding rainfall affecting the recharge in the current period and α^i is a row matrix containing number of parameters. In the procedure for soil moisture budgeting (Eqn.2.7), the parameter determined will be the field capacity. In the absence of the availability of the bulk data, the simple functional form that can be used for the estimation of the built-in parameters is

$$R_{ik} = K_{r_{ik}} P_{ik} \quad \ldots\ldots \quad (4.9)$$

where $K_{r_{ik}}$ is the recharge coefficient and P_{ik} is the precipitation. It varies from basin to basin, and for any given basin, a temporal variability of $K_{r_{ik}}$ exists. For shorter time periods, the antecedent soil moisture conditions can be incorporated through a parameter threshold rainfall (α_3^i) which is the minimum rainfall required in one time period to fill the soil moisture deficiency. Thus, rainfall recharge relation can be written as

$$R_{ik} = \alpha_1^i \left[\alpha_2^i P_{i,k} + \left(1 - \alpha_2^i\right) P_{i,k-1} - \alpha_3^i \right] \quad \text{if} \quad P_{i,k-1} < \alpha_3^i$$

$$= 0 \quad \text{if } \alpha_3^i > P_{i,k} + \left(1 - \alpha_2^i \right) P_{i,k-1} \text{ and } P_{i,k-1} < \alpha_3^i$$

...... (4.10)

$$\text{Now} \quad R_{ik} = \alpha_1^i \left[\alpha_2^i P_{i,k} + \left(1 - \alpha_2^i \right) P_{i,k-1} \right] \quad \text{if } P_{i,k-1} \geq \alpha_3^i$$

..... (4.11)

where α_1^i, α_2^i and α_3^i are recharge parameters of the i^{th} point. The α_1^i corresponds to the recharge coefficient and α_2^i accounts for the delayed aquifer response. The approach for incorporating the rainfall recharge in the inverse problem formulation not only provides consistent estimates of historical rainfall recharge, but also effectively calibrates the locally acceptable functional form of the rainfall recharge relations (Kashyap, 1981).

The recharge due to canal seepage can be calculated from the expression (modified form of Eqn.4.3)

$$C_{ik} = I_c + I_r + I_i + I_w - D_0 \qquad \text{....... (4.12)}$$

where I_c is recharge due to percolation from main canal and branches, I_r is the recharge due to percolation from reaches, I_i is the percolation from canal irrigated areas, I_w is percolation water released from areas irrigated by wells and D_0 is the discharge from canals, branches and reaches

across the boundary of basin to outside of system under study. Thus, total loss of groundwater to the rivers as subsurface outflow from the area is also included in this component D_0.

4.4 ESTIMATION OF THE GROUNDWATER SYSTEM ELEMENTS

As indicted earlier, (Chapter 3), the approximating least square function is employed for obtaining system elements relating to the groundwater studies.

4.4.1 Groundwater Storage

The groundwater storage in k^{th} time can be estimated by integrating the function $H_k(p,q)$ over the space defined by the study area. Thus, the groundwater storage SG above a datum plane ZD is given by

$$SG_k = \iint_{AG} S\left[H_k\,(p,q) - ZD\right] dp\, dq \qquad \ldots.\ (4.13)$$

where AG is the study area. The integration over an irregularly shaped area can be carried out by dividing the area into finite number of strips extending along one of the two directions p or q. The groundwater storage in the i^{th} strip (Fig.4.1) is given by

$$\Delta S_i = \int_{p=p_i^m}^{p_i^n} \int_{q=q_i^m}^{q_i^{m+\Delta q}} S \left[H_k\,(p,q) - ZD \right] dp\, dq \qquad \ldots\ldots\ldots(4.14)$$

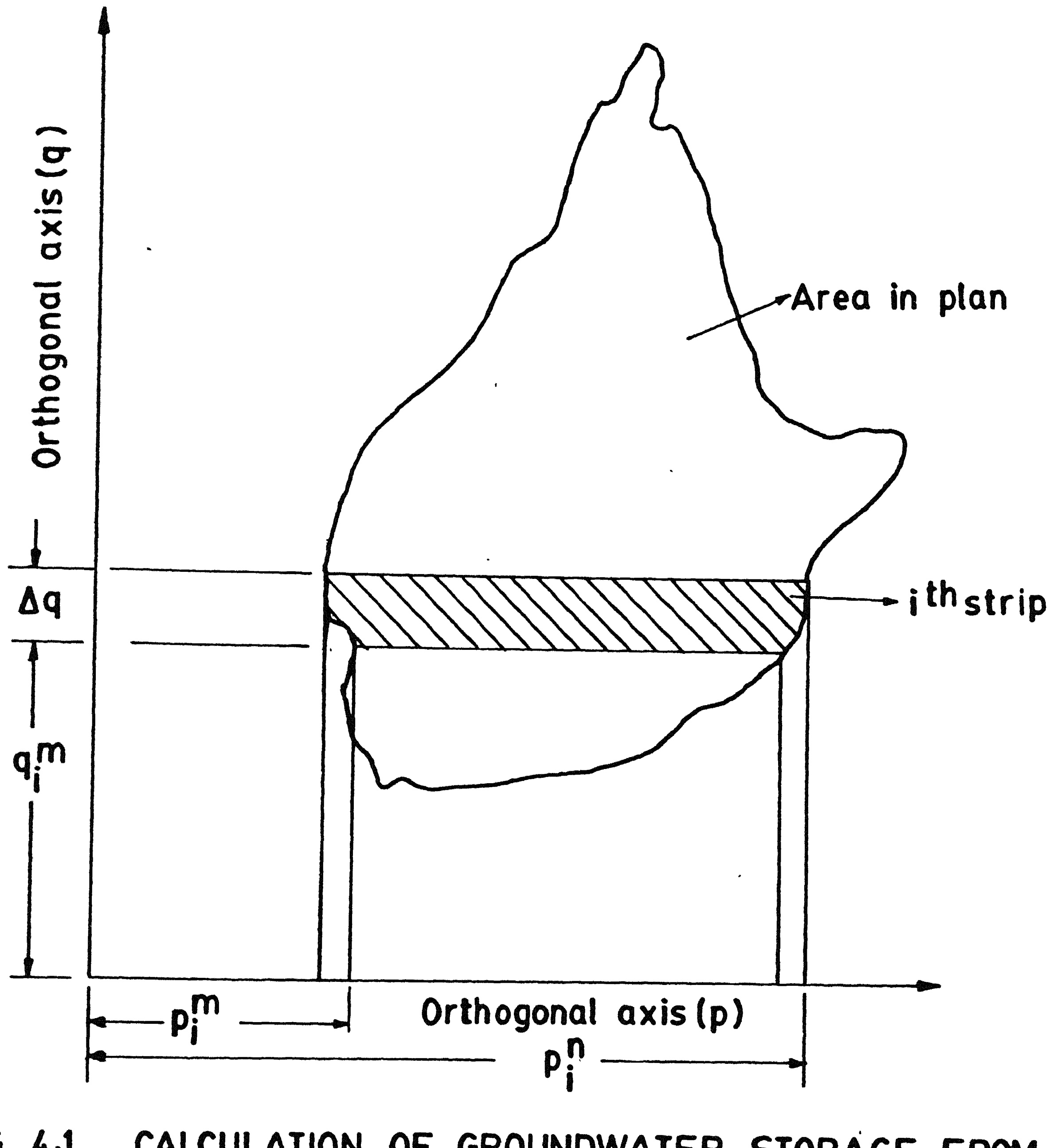


FIG. 4.1 CALCULATION OF GROUNDWATER STORAGE FROM DERIVATIVES OF HEAD

Total groundwater storage is given by

$$SG_k = \sum_{i=1,n}^{ns} \Delta S_i \qquad \ldots\ldots (4.15)$$

where ns is the number of strips. The function [H_k (p,q) - ZD] can be integrated analytically. Thus, ΔS_i can be evaluated directly if the storativity S is uniform in i^{th} strip. If S is varying in i^{th} strip, then SG may have to be evaluated by the numerical solution of equation (4.15).

4.4.2 Boundary Recharge

Total subsurface inflow rate (I_k) across the boundary of the an area at k^{th} time point is written as

$$I_k = \int_c - \frac{\partial}{\partial n^c} H_k(p,q)\ T\ dc \qquad \ldots. (4.16)$$

where, c is boundary contour and n^c is its inner normal. The integration can be performed by dividing the boundary contour into finite number of elementary lengths (Fig.4.2). The subsurface flow over i^{th} elementary length is given by

$$\Delta\ I_i = \frac{\partial}{\partial n^c} H_k\ (p_i^*, q_i^*)\ \Delta L_i T_i \qquad \ldots..(4.17)$$

where p_i, q_i are the two coordinates of the central point of i^{th} elementary length and T_i is the average transmissivity at its elementary length (ΔL_i).

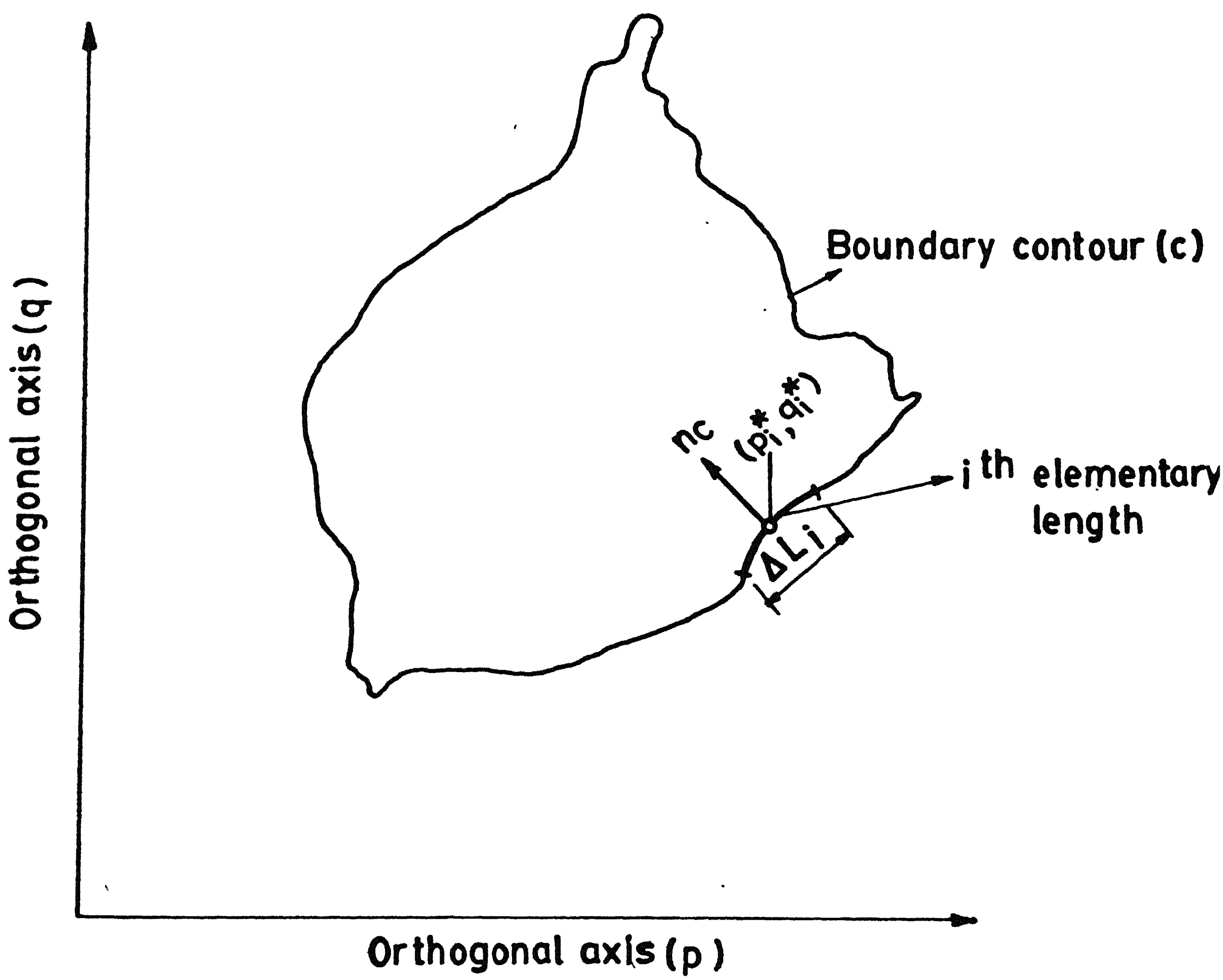


FIG. 4·2 ESTIMATION OF LATERAL FLOWS ACROSS THE BOUNDARY

$$I_k = \sum_{i=1}^{ne} \Delta \ I_i \qquad \ldots\ldots (4.18)$$

where ne is the total number of elementary lengths. In equation 4.18, the hydraulic gradient in the direction of n^c can be evaluated directly, by knowing the average orientation of the elementary length with respect to p and q directions.

In the present study,the polynomial coefficients are calculated first and and then groundwater storage and boundary recharge are estimated using the equations 4.14 to 4.18.

4.5 ESTIMATION OF RECHARGE COMPONENTS IN THE STUDY AREA

For the study area, the recharge has been estimated on the basis of data pertaining to rainfall and canal seepage, and discharge values from well draft and evaporation factors. The total algebraic sum of inflows and outflows (Q_{ik}) at i^{th} space point and k^{th} period is expressed as

$$Q_{ik} = R_{ik} - W_{ik} + C_{ik} + X_{ik} \qquad \ldots\ldots (4.19)$$

The matrix R_{ik} indicating recharge due to rainfall is calculated by the equation 4.19. In this equation, P_{ik} is the precipitation during k^{th} time period and at i^{th} space point, and the data for the same were collected from Andhra Pradesh State Groundwater Department. There are only two rain gauge

estimates. The matrix W_{ik} (in Eqn.4.19) representing effective withdrawl due to pumpage. The draft from each individual well is calculated using the norms prescribed by Andhra Pradesh Sate ground Water Department. The discharge values considered from a tube well and from a dug well are $35000 m^3$/day and $20000 m^3$/day respectively (including the effect of other pumping wells which are not observation wells). The effective withdrawl values due to pumpage (W_{ik}) were calcuglated using the above norms at 69 observation wells and for 120 months. Values of $\bar{W}_{ik}$ for the month of March, 81 are presented in Table 4.1.

The term C_{ik} in Eqn.4.19 represents recharge through seepage from canals and reaches. Estimates of total seepage were obtained taking into consideration conveyance losses for the main canal and branch canals as 1.858 million cubic meters and 1.1148 thousand cubic meters respectively, based on the recommendation of George Smout, World Bank expert (A.P.S.G. W.D. Report, 1981). The actual canal discharge values (in m^3/month) for each canal have been calculated on the basis of the total discharge data (as obtained from Andhra Pradesh Irrigation Department, Canals Office of Nagarjuna Sagar Left Canal Command area, Khammam, A.P) and the total number of running days. The recharge due to canal seepage for all the 120 months at 69 observation points were estimated depending on their distance from the canals and/or reaches and and the polygonal area of influence for the well. Estimates of C_{ik} for

the month of March,`81 are also presented (Table 4.1).

All the other recharge and discharge components are taken care of by the term X_{ik} (Eqn.4.19) where losses due to evaporation and irrigation effects on canal recharge etc., are included in the matrix. The value for the evaporation, taken as 0.987E-07m/sec, is based on the monthly evaporation data obtained from A.P.S.G.W. Department. The transpiration effect was not seperately considered. The irrigation effect is taken as 5.0% of discharge running in the canals and it is equally distributed to all the observation points. These values at all 69 observation points and for all the 120 time periods are also calculated. The values for a typical month are shown in Table 4.1.

The calculated recharge and discharge values from various factors such as rainfall, canal seepage, irrigation effect, groundwater withdrawl and evaporation are stored for use in the estimation of aquifer parameters (Chapter 5).

Table 4.1: Recharge Values for a Typical Month (March, 1981)

(in thousand cubic meters)

Observation well No.	R 1	C 2	W 3	X 4	Q 1+2-(3+4)
1.	55.58	184.09	20	25	194.67
2.	66.69	184.09	35	25	190.78
3.	116.71	479.68	20	25	151.39
4.	63.92	479.68	20	25	498.60
5.	247.33	783.06	20	25	985.39
6.	77.01	303.37	20	25	326.18
7.	61.14	11774.42	20	25	11790.56
8.	86.14	184.09	35	25	210.24
9.	63.92	184.09	20	25	203.01
10.	61.14	479.68	20	25	495.82
11.	18.53	303.37	20	25	276.90
12.	17.37	274.85	20	25	247.22
13.	69.47	36.30	35	25	45.77
14.	80.58	36.30	20	25	71.89
15.	111.16	11738.17	20	25	11804.33
16.	50.02	184.09	35	25	174.11
17.	66.69	479.68	20	25	541.38
18.	34.09	479.68	20	25	458.77
19.	8.68	1685.39	20	25	1649.07
20.	11.58	324.11	20	25	290.69
21.	20.26	342.26	20	25	317.52
22.	24.90	67.46	20	25	47.36
23.	23.16	95.53	20	25	73.69
24.	37.05	11738.17	20	25	11730.22
25.	19.11	11738.17	35	25	11697.28
26.	31.84	11738.17	20	25	11725.01
27.	4.05	11738.17	20	25	11697.22
28.	11.58	1037.17	20	25	1003.75
29.	22.00	560.07	20	25	537.07
30.	20.26	95.53	20	25	70.79
31.	2.89	95.53	35	25	38.42
32.	5.21	95.53	20	25	55.74
33.	8.68	11738.17	20	25	11701.85
34.	8.68	11738.17	20	25	11701.85
35.	21.42	11738.17	35	25	11699.59
36.	21.42	342.26	20	25	318.68
37.	32.42	313.74	20	25	301.16
38.	24.32	344.86	20	25	344.18
39.	12.74	11738.17	35	25	11690.91
40.	9.84	11738.17	20	25	11703.01

..... Contd......

Table 4.1 Contd....

Observation well No.	R 1	C 2	W 3	X 4	Q 1+2-(3+4)
41.	20.84	11738.17	35	25	11699.01
42.	5.79	342.26	20	25	303.05
43.	12.74	342.26	20	25	310.00
44.	16.21	342.26	20	25	313.47
45.	19.68	95.53	20	25	115.21
46.	12.16	440.80	20	25	407.96
47.	17.95	11738.17	20	25	11711.12
48.	24.32	11738.17	35	25	11702.49
49.	30.69	11738.17	20	25	11723.86
50.	42.26	11738.17	20	25	11735.43
51.	37.05	11738.17	35	25	11715.22
52.	13.89	342.26	20	25	311.16
53.	9.84	342.26	20	25	307.10
54.	13.32	342.26	20	25	310.58
55.	11.58	342.26	20	25	308.84
56.	20.84	11738.17	20	25	11714.01
57.	22.00	11738.17	20	25	11715.17
58.	15.63	11738.17	35	25	11693.80
59.	21.42	11738.17	35	25	11699.59
60.	30.11	11738.17	35	25	11708.28
61	39.95	342.26	35	25	322.21
62.	23.74	324.26	20	25	302.99
63.	47.48	11738.17	20	25	11740.68
64.	32.42	11738.17	35	25	11710.59
65.	63.69	11738.17	35	25	11742.86
66.	41.69	11738.17	20	25	11738.85
67.	23.74	11738.17	20	25	11716.91
68.	21.42	11738.17	20	25	11714.59
69.	24.89	11738.17	35	25	11703.07

R : Recharge due to rainfall

C : Recharge due to canal seepage

W : Discharge due to well draft

X : Discharge due to evaporation and irrigation effect

Q : Net inflows and outflows

CHAPTER 5

ESTIMATION OF AQUIFER PARAMETERS BY INVERSE PROBLEM

5.1 INTRODUCTION

By known transmissivity (T) and storativity (S),spatially distributed over the problem domain, estimation of water levels by simulating the aquifer response to a deterministic pattern of ground water withdrawals and recharge is known as 'direct problem' in groundwater hydrology. Pump test, a popular means of estimating aquifer parameters (T and S), involves generating the aquifer response to the pumping in a single well. In the direct problem, T and S values, as obtained from pump tests are assigned to nodal points in a distributed model. However, these values are approximate as the pump test data analysis is based on several assumptions such as isotropic aquifer with a fully-penetrating well of infinitesimal size. Further spatial limitation of test pump test sites on a regional basis exisists due to the high cost of operations.

On the other hand, the 'inverse problem' is an approach to estimate T and S by employing historical data of aquifer response and corresponding aquifer excitations. The aquifer

excitation can be either of the form of vertical accretions (Kleinecke, 1971; Sagar et al, 1973, 1975) or change in the boundary conditions like river stage (Singh and Sagar, 1977).

In this chapter, a numerical scheme for estimating aquifer parameters has been developed based on the analysis of historical record of aquifer response to the vertical accretions such as pumping, rainfall recharge and canal seepage. The principal permeability directions and coefficient of recharge due to rainfall can also be estimated directly along with the aquifer parameters (T and S) using the inverse problem method.

5.2 THE GOVERNING DIFFERENTIAL EQUATION

The governing differential equation of two-dimensional transient ground water flow in a heterogeneous and isotropic confined aquifer can also be written as

$$T_{xx} \frac{\partial^2 h}{\partial x^2} + T_{yy} \frac{\partial^2 h}{\partial y^2} + \frac{\partial T_{xx}}{\partial x} \frac{\partial h}{\partial x} + \frac{\partial T_{yy}}{\partial y} \frac{\partial h}{\partial y} + Q = S \frac{\partial h}{\partial t} \qquad \text{...... (5.1)}$$

This equation is strictly valid for only confined aquifers as it assumes a time-invariant saturated flow and uniform horizontal velocity over the entire depth of the flow. However, this equation can be applied for unconfined aquifers

if the following assumptions are satisfied.

(1) Dupit-Forchheimer's assumptions namely

(a) the velocity of the flow to be proportional to the tangent of the hydraulic gradient instead of the sine as defined by Darcy's equation, and

(b) the flow to be horizontal and uniform everywhere in a vertical section (Todd, 1980).

(2) Temporal fluctuations of water table, are small in-comparison to the mean saturated thickness.

Assuming the hydraulic gradients $\left[\frac{\partial h}{\partial x}, \frac{\partial h}{\partial y}\right]$ and transmissivity gradients $\left[\frac{\partial T_{xx}}{\partial x}, \frac{\partial T_{yy}}{\partial y}\right]$ to be small, their products (Eqn.5.1) will be negligibly smaller in comparison to the other terms. Thus, neglecting the product terms, Eqn. 5.1 reduces to the following form

$$T_{xx} \frac{\partial^2 h}{\partial x^2} + T_{yy} \frac{\partial^2 h}{\partial y^2} + Q = S \frac{\partial h}{\partial t} \qquad .... (5.2)$$

where $\frac{\partial^2 h}{\partial x^2}$ and $\frac{\partial^2 h}{\partial y^2}$ are second spatial derivatives of the piezometric head in the directions of principal permeabilities.These can be derived in terms of their respective derivatives D_p and D_q in any two arbitrarily chosen orthogonal directions p and q as

$$\frac{\partial^2 h}{\partial x^2} = D_p \cos^2\theta + D_q \sin^2\theta \qquad ...... (5.3)$$

and $$\frac{\partial^2 h}{\partial y^2} = D_p \sin^2\theta + D_q \cos^2\theta \qquad \ldots\ldots (5.4)$$

In the above assumptions, the terms containing $\frac{\partial h}{\partial p}\frac{\partial \theta}{\partial x}$, $\frac{\partial h}{\partial q}\frac{\partial \theta}{\partial x}$, $\frac{\partial h}{\partial p}\frac{\partial \theta}{\partial y}$ and $\frac{\partial h}{\partial q}\frac{\partial \theta}{\partial y}$ have been neglected, because of their small order of magnitude. As shown in Fig.5.1 θ is the angle between two sets of orthogonal axes. Sagar (1975) presented this governing equation in terms of T_{pp}, T_{qq} and T_{pq} with the associated inequality constraint $T_{pp} + T_{qq} - (T_{pq})^2 \geq 0$. In the present approach, this constraint is eliminated and for the explicit estimation of the orientation of the principal permeability directions, equation 5.2 can be expressed in the form

$$T_{xx}\left[D_p \cos^2\theta + D_q \sin^2\theta \right] + \left[D_p \sin^2\theta + D_q \cos^2\theta \right] T_{yy} + Q = S \frac{\partial h}{\partial t} \qquad \ldots\ldots (5.5)$$

For an anisotropic aquifer with spatial variation, the solution is obtained using the 'direct approach' (Neuman, 1975) by solving this governing differential equation using finite element methods (Freeze and Cherry, 1979; Pinder and Gray, 1977).

5.3 DISCRETISATION OF THE GOVERNING DIFFERENTIAL EQUATION

Equation 5.5 is the continuous form of the governing differential equation. Its discretisation involves writing

down the following terms for discreet space and time points.

i) Recharge (Q)

ii) Spatial and temporal derivatives $\left[\frac{\partial^2 h}{\partial p^2}, \frac{\partial^2 h}{\partial q^2} \text{ and } \frac{\partial h}{\partial t}\right]$

iii) Aquifer parameters (T and S)

Employing the discretised form of Q as obtained from Eqns.4.11 and 4.12, equation 5.5 can be expressed as

$$T_{xi}\left[DP_{ik}\cos^2\theta + DQ_{ik}\sin^2\theta\right] + T_{yi}\left[DP_{ik}\sin^2\theta + DQ_{ik}\cos^2\theta\right] +$$

$$f_r\left[P_{i,k}, P_{i,k-1}, \ldots\ldots\ldots, P_{i,k-me}, \alpha^i\right] +$$

$$C_{ik} + X_{ik} - W_{ik} - S_i\, DT_{ik} = 0$$

$$\ldots\ldots (5.6)$$

where T_{xi}, T_{yi} and S_i are aquifer parameters at i^{th} space point corresponding to T_{xx}, T_{yy} and S. And DP_{ik}, DQ_k and DT_{ik} are the discretised forms of DP, DQ and $\frac{\partial h}{\partial t}$ for i^{th} space point at k^{th} time point. Eqns. 5.12 and 5.13 can be written as

$$f\left[T_{xi}, T_{yi}, \theta_i, S_i, \alpha^i, f_r, DP_{ik}, DQ_{ik}, DT_{ik}, W_{ik}, X_{ik}, C_{ik},\right.$$

$$\left.P_{i,k}, P_{i,k-1}, \ldots, P_{i,k-me}\right] = 0 \qquad \ldots\ldots\ldots (5.7)$$

Equations 5.6 and 5.7 express the governing differential equation in terms of

i) System properties

T_{xi}, T_{yi}, S_i, θ_i, α^i, f_r

ii) Information derived from rainfall records

$P_{i,k}, P_{i,k-1}, \ldots, P_{i,k-me}$

iii) Information derived from piezometric head data

$\left[DP_{ik}\right], \left[DQ_{ik}\right], \left[DT_{ik}\right]$

iv) Information derived from hydrologic, agricultural and irrigation records

$\left[W_{ik}\right], \left[C_{ik}\right], \left[X_{ik}\right]$

5.4 ESTIMATION OF DERIVATIVES

The terms DP_{ik} and DQ_{ik} appearing in equations 5.6 and 5.7 are estimates of the water table head in any two arbitrarily decided orthogonal directions p and q, at an angle θ with principal permeability directions (p'and q') as indicated in Fig.5.1 and the term DT_{ik} is the first temporal derivative. These derivatives can be directly obtained by differentiating the approxomating polynomial twice. Sagar et al. (1973, 1975) have proved that second spatial derivatives are necessary for estimating trensmissivity by direct methods of inverse problem. The least square polynomial apart from smoothening the data, also provides a closed form differentiable functional relationship between the water table head and the spatial coordinates. Thus, if $H_k(p,q)$ and $H_{k+1}(p,q)$ are the least square functional relations between

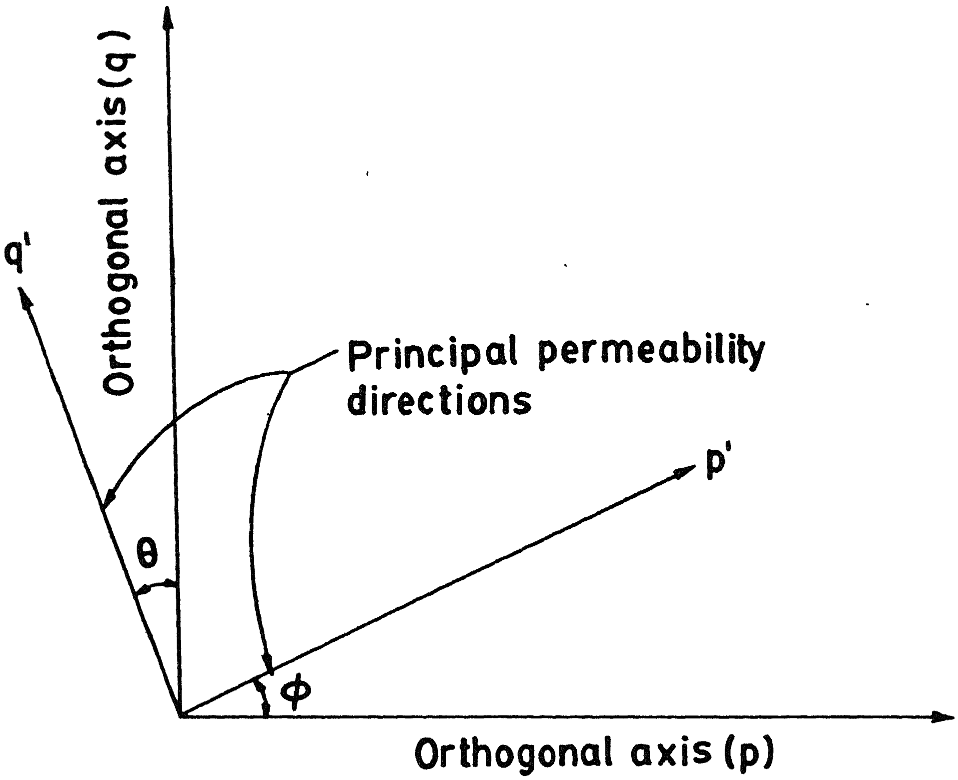


FIG. 5.1 DIRECTIONS OF PRINCIPAL PERMEABILITIES

the water table head and the space coordinates p and q, at the beginning and end of the k^{th} time interval, then

$$DP_{ik} = \frac{1}{2}\left[\frac{\partial^2}{\partial p^2} H_k\left(p_i,q_i\right) + \frac{\partial^2}{\partial p^2} H_{k+1}\left(p_i,q_i\right) \right] \quad .... (5.8)$$

$$DQ_{ik} = \frac{1}{2}\left[\frac{\partial^2}{\partial q^2} H_k\left(p_i,q_i\right) + \frac{\partial^2}{\partial q^2} H_{k+1}\left(p_i,q_i\right) \right] \quad .... (5.9)$$

$$DT_{ik} = \frac{H_{k+1}\ (p_i,q_i) - H_k\ (p_i,q_i)}{\Delta t_k} \quad .... (5.10)$$

where (p_i,q_i) are the coordinates (p,q) of the i^{th} space point and Δt_k is the span of the k^{th} interval. Equation 5.6 is the trigonoalgebraic form of Eqn.5.2. The estimation of the derivatives (DP_{ik}), (DQ_{ik}), and (DT_{ik}) can be made from historical water table data, and the estimates of (W_{ik}), (C_{ik}), and (X_{ik}) from available historical data relating to the groundwater withdrawals, canal seepage, geology, evapotran- spiration etc.. Substitution of these estimates in Eqn.5.6 for any given values of i and k, yields an equation in terms of system parameters. The equation thus evolved is indeterminate as it involves (4+nc) unknown terms. However, it can be converted to a deterministic one, when designed to estimate the properties of single space point which will consist of Eqn.5.6 written for i^{th} space point and (4+nc)

different time periods i.e.,

$$T_{xi}\left[DP_{ik} \cos^2\theta + DQ_{ik} \sin^2\theta\right] + T_{yi}\left[DP_{ik} \sin^2\theta + DQ_{ik} \cos^2\theta\right] +$$

$$f_r \left[P_{i,k}, P_{i,k-1}, \ldots\ldots\ldots, P_{i,k-me}, \alpha^i\right] +$$

$$- S_i \; DT_{ik} = C_{ik} + X_{ik} - W_{ik}$$

$$\ldots\ldots \quad (5.11)$$

where k = 1.......(4+nc) and nc is the number of rainfall-recharge parameters.

The resulting system of non-linear simultaneous equations can be solved for (4+nc) unknown variables. This procedure can lead to many problems, since there are no fool-proof methods for the solution of non-linear simultaneous equations. The system of equations can be converted into a linear form by assuming linear rainfall relation and assigning some constant value to α_1^i (from the known hydrogeological condition of the aquifer under study).

5.5 OPTIMIZATION

Equation 5.6 is the discretised form of Eqn.5.2 and in addition, it incorporates the functional relationship for rainfall-recharge. The right hand side of this equation may not be exactly equal to zero under the actual field conditions, due to any of the following reasons :

1) There are many built-in assumptions in Eqn.5.2 which may not be always true. Neuman (1973) has pointed out large number of physical situations which may violate many of these assumptions. These physical situations include existence of fractures and dykes, unsaturated flow, compressibility, three- dimensional geometry, changes in the fluid density, non- Darcian flow in the areas of high Reynolds' number.

2) Numerical errors associated with the estimation of spatial and temporal derivatives of water table head i.e. (DP_{ik}),(DQ_{ik}) and (DT_{ik}). These errors can originate from the numerical algorithms adopted to arrive at the derivatives.

3) The assumptions incorporated in the adopted rainfall-recharge relation (Eqns. 4.8, 4.9 and 4.10).

4) The assumptions involved in calculating seepage from canals (the constants in the empirical formulae) may not exactly represent the study area.

Because of these possible discrepancies, the solution of simultaneous equation may yield grossly misleading values of the parameters, and can not be used to estimate the parameters in most of the real world situations (Kashyap, 1981).

Keeping this in view, equation 5.6 can be expressed as

$$T_{xi}\left[DP_{ik} \cos^2\theta + DQ_{ik} \sin^2\theta\right] + T_{yi}\left[DP_{ik} \sin^2\theta + DQ_{ik} \cos^2\theta\right] +$$

$$f_r \left[P_{i,k}, P_{i,k-1}, \ldots\ldots\ldots, P_{i,k-me}, \alpha^i\right] +$$

$$C_{ik} + X_{ik} - W_{ik} - S_i\ DT_{ik} = E_{ik}$$

$$\ldots\ldots (5.12)$$

where E_{ik} is the residue of the governing differential equation when written in the discrete form as given in equation 5.6, for the i^{th} space point and k^{th} time period. The exact magnitudes of (E_{ik}) are not known. However, these residues will have certain frequency distribution. One of the most obvious frequency distribution for large sample is normal distribution with zero mean. The frequency distribution could be written as

$$f(\xi) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[- \frac{1}{2} \left[\frac{\xi}{\sigma}\right]^2 \right] \quad \ldots\ldots (5.13)$$

where σ is standard deviation and ξ is frequency distribution.

If this assumption relating to the normal distribution of the residues holds good, then the minimization of the sum of the squares of residues yield the most likelihood estimation i.e. minimize Y given by

$$Y_1 = \sum_{i=1}^{n} \sum_{k=1}^{m} E_{ik}^2 \qquad \ldots . (5.14)$$

The above distribution is valid only if sample size is large and there are no systematic errors. The other commonly used residue functional is the sum of moduli of the residues. The minimization of functional yields the most likelihood solution if the residues are distributed as for the following distribution:

$$f(\xi) = \frac{1}{2\pi\sigma} \quad \exp\left[\frac{|\xi|}{\alpha_p}\right] \qquad \ldots . (5.15)$$

where α_p is confidence level at p.

Now the most likelihood estimates correspond to

$$\text{minimize} \quad \sum_{i=1}^{n} \sum_{k=1}^{m} |E_{ik}| \qquad \ldots\ldots (5.16)$$

The objective function to be minimized can incorporate residue function by minimizing either squares of the residue or modulus of the residue, which are summed up over time-domain or over both time and space domains. Thus, the objective function can assume one of the following forms, apart from those given in Eqns.5.14 and 5.16.

$$Y_2 = \sum_{k=1}^{m} E_{ik}^2 \qquad \ldots\ldots (5.17)$$

$$Y_2 = \sum_{k=1}^{m} |E_{ik}| \qquad \text{for a given } i,\ i = 1,2,..n$$

where n is total number of space points for which the parameters are to be estimated and m is the total number of time periods for which the data are available.

Y_2 is a function of aquifer parameters and hydrogeological properties (i.e. the system parameters) of the i^{th} space point under consideration. The total number of decision variables involved in the minimization of Y_2 equals the number of time periods i.e. k = 1.....m. Thus, in each separate call to the optimization routine, aquifer parameters for one space point are estimated for different time periods (k). This approach allows number of decision variables to a manageable limit, but does not permit the inclusion of constraints relating to the spatial variation of the aquifer parameters. This type of constraints may be necessary from the view point of getting a smooth solution (Emsellem and deMarsily, 1971). These can be incorporated in the estimation of parameters provided the parameters of all the space points are estimated by the minimization of single objective function. This is permitted by minimization of Y_1 (Eqns. 5.14 and 5.15) which is function of aquifer parameters at all the space points. The total number of decision variables involved in the minimization goes up to (n X m) times for Y_1. This increase in turn increases uncertainties generally associated with non-linear programming.

5.5.1 Constraints

The actual frequency distribution of the residues may follow the general pattern of the normal or exponential distribution, but may not strictly follow the geometrical relation as stipulated by the distributions (Eqns. 5.13 and 5.15). As a result, the residues corresponding to the minimized residue functionals will seldom be identical to the actual residues. Thus, there may be an infinite number of near-optimal solutions which may vary considerably from the optimal solution but may lead to a much better predictive model (Neuman, 1973). In these cases, the knowledge relating to the geohydrology of the aquifer helps to choose the best set of parameters for obtaining the near-optimal solutions. These values of the parameters with judicious quantification can be incorporated in the parameter estimation program in the form of appropriate constraints which apart from ensuring non-negativity of parameters where required, may also stipulate the permissible range of variation or each of few of the constraints. The stipulated ranges of constraints may be more restrictive in nature, as the range is fixed on the basis of theoretical conditions. Similarly constraints can be imposed to restrict the spatial variation of aquifer parameters and to obtain a 'smooth solution' (Emsellem and deMarsily, 1971).

5.5.2 Sequential Unconstrained Minimization Technique

The algorithm that can be employed to arrive at the optimal solution depends on the nature of the residue functional and constraints. The unconstrained minimization of the sum of squares of the residues can be accomplished by classical least square approach. This approach, widely used, in general involves a set of linear simultaneous equations. The constrained minimization of the sum of the squares of the residues or the sum of moduli of the residue can be carried out employing, with necessary modifications, one of the standard non-linear optimization algorithms. In the present work Sequential Unconstrained Minimization Technique, SUMT (McCormic and Fiacco, 1968) has been used for the same. This program finds the minimum of a multivariable non-linear function subject to inequality and equality constraints.

The SUMT uses the problem constraints and the original objective function which is minimized by any appropriate unconstrained multivariable technique. The steps involved in the algorithm are as follows :

1. A modified objective function is formulated consisting of the original function and penalty function with the form where r is a positive constant. As the algorithm progresses, r is revaluated to form a monotonically decreasing sequence $r_1 > r_2 > r_3$.... As r becomes small under suitable conditions, P approaches F and the problem is solved.

2. Select a starting point (feasible or nonfeasible) and an initial value of r.
3. Determine the minimum of the modified objective function for the current value of r using SUMT or any appropriate technique.
4. Estimate the optimal solution using extrapolation fromulae.
5. Select a new value of r and repeat the procedure until convergence criteria are satisfied.

This program consists of a main routine (to control subroutines that do special purpose of SUMT) and four user supplied subroutines.

5.6 ESTIMATION OF AQUIFER PARAMETERS

Aquifer parameters for the study area are estimated by considering the data of 20 discrete sequential periods of six months duration each. This was selected based on easy water budgeting nature in it i.e., when a water year is divided into two parts, each part has its own water-balancing nature and hence is a relatively closed system to operate. In the present study, the periods from December to May and from June to November are taken as pre-monsoon and post-monsoon periods respectively. Accordingly, the 66 months between June '78 and November' 83 have been divided into 11 slabs of six months each in the first five year set. The last six months period is taken as the overlap period (Table 5.1).

Table 5.1 : Slabs for Parameter Estimation (Set 1)

sl.no.	period	slab no.
1.	Jun.'78 to Nov.'78	1
2.	Dec.'78 to May.'79	2
3.	Jun.'79 to Nov.'79	3
4.	Dec.'79 to May.'80	4
5.	Jun.'80 to Nov.'80	5
6.	Dec.'80 to May.'81	6
7.	Jun.'81 to Nov.'81	7
8.	Dec.'81 to May.'82	8
9.	Jun.'82 to Nov.'82	9
10.	Dec.'82 to May.'83	10
11.	Jun.'83 to Nov.'83	Overlap period

The following data are considered for the modelling :

a) The coefficients of the least square polynomial approximating for each of the periods which are calculated for all the sixty six months as explained in Chapter 3 and derivatives are calculated (as detailed in Section 5.4). These derivatives are stored as column matrices $\left[DP_{ik}\right]$, $\left[DQ_{ik}\right]$ and $\left[DT_{ik}\right]$ for i^{th} space point.

b) The rainfall figures for all the rain gauge stations, during each k^{th} period and stored as a column matrix $\left[P_{ik}\right]$ for all space points. The rainfall distribution graph is presented in Fig.1.2.

c) The effective ground water recharge and withdrawal values at each observation point(i), and for each of the periods,k, as $\left[C_{ik}\right]$, $\left[X_{ik}\right]$ and $\left[W_{ik}\right]$.

These data were employed to generate the following column matrices for a given i^{th} space point.

$$\left[DP_{ik}\right],\ \left[DQ_{ik}\right],\ \left[DT_{ik}\right],\ \left[P_{ik}\right],\ \left[C_{ik}\right],\ \left[X_{ik}\right] \text{ and } \left[W_{ik}\right]$$

Each matrix has 11 elements corresponding to the 11 periods. Marginal evapotranspiration values near the river and the canal banks have been incorporated into $\left[W_{ik}\right]$. The following initial values are adopted for the decision variables.

$$S_i = 0.2000,\ T_{xi} = 150.00\ m^2/day,\ T_{yi} = 150.00 m^2/day$$

$$\theta_i = 0.00\ ,\ \alpha_1^i = 0.15\ mm/month,\ \alpha_2^i = 0.85 mm/month$$

and $$\alpha_3^i = 20 mm/month \qquad \ldots (5.18)$$

The initial values of T_{xi}, T_{yi} and S_i are taken from the pump test data. The initial value of α_2^i is assigned assuming a 15% carry-over effect from the previous period. The decision regarding initial value of θ, α_1^i and α_3^i were made depending on the basin characteristics.

By the above setup, the residues of the discretised Boussinesq's equation (5.11) for the i^{th} space point corresponding to all the eleven periods are completely described by the input data matrices and the initial values of the seven decision variables. For the estimation of optimal parameters, the following residue functional is adopted.

$$Y_i = \sum_{k=1}^{11} E_{ik}^2 \qquad \ldots\ldots (5.19)$$

Where $$Y_i = Y \left[S_i,\ T_{xi},\ T_{yi},\ \theta_i,\ \alpha_1^i,\ \alpha_2^i,\ \alpha_3^i \right] \qquad \ldots\ldots (5.20)$$

The minimization of Y_i with respect to these seven decision variables is carried out subject to the following constraints.

$$00.30 > S_i > 00.03$$
$$5000 > T_{xi} > 40.00$$
$$5000 > T_{yi} > 40.00$$
$$\pi/2 > \theta_i > 00.00$$
$$00.20 > \alpha_1^i > 00.00$$
$$01.00 > \alpha_2^i > 00.80$$
$$80.00 > \alpha_3^i > 00.00$$

Total number of constraints are fourteen.

In the trial runs, aquifer parameters are estimated at 30 space points by minimizing Y_i subject to the above constraints, using a non-linear optimization technique (SUMT). The results are far from the actual field values for all the space points, since maximum number of S values (20 out 30) are touching the lower limit (0.03) and values of T are closer to the upper limit (5000 m^2/day). Even when the initial value of S is reduced to 0.00, all the estimates of S were still closer to the lower limit. At this stage where all initial values and constraints were rechecked and the entire data were thoroughly examined, it was observed that net inflows $\left[C_{ik}+X_{ik}\right]$ are far greater than net outflows $\left[W_{ik}\right]$, the excess of $\left[C_{ik}+X_{ik}\right]$ over $\left[W_{ik}\right]$ is caused due to additional contribution to storage $\left[-S_i DT_{ik}\right]$ and the net inflows $\left[X_{ik}+C_{ik}\right]$ are directly related to the estimation of higher values of T_{xi} and T_{yi}. This problem can be solved by writing the equation in the form

$$C_{ik} + X_{ik} - W_{ik} = \left[- S_i \; DT_{ik} \right] + \left[T_{xi} \left(DP_{ik} \cos^2\theta_i + DQ_{ik} \sin^2\theta_i \right) + T_{yi} \left(DP_{ik} \sin^2\theta_i + DQ_{ik} \cos^2\theta_i \right) \right] \qquad \text{....... (5.21)}$$

The excess values of $\left[C_{ik}+X_{ik}-W_{ik}\right]$ in the optimal solution are assigned to the storage term at each space point. It may be indicated here that the estimation of the parameters is influenced by the initial values assigned to the each of the space points. Hence, the aquifer parameters had to be estimated through trial and error by assigning initial values to all the 69 space points and checking the estimates.

To solve this problem, a sequence of space points are estimated one after another by considering the nearest space point as a next point to be estimated from the first point. This scheme also did not give any improvement in the estimates. So another scheme was adopted, where, the estimated values at the first spatial point are given as initial values to the nearest space point. As a cross check, the estimated parameters at selected space points are compared with the actual values obtained through pump tests at the respective points. It has been observed that both the values are in agreement within 10 percent range. The estimated transmissivity values at the end of May 1983 are presented in Table 5.2. The storativity values for all the space points are

Table 5.2 Transmissivity and Storage Estimates (Set 1)

well No.	transmissivity sq.m/day	storage (MCM)
1	56.84	30.39
2	171.65	35.02
3	59.10	66.26
4	128.19	35.62
5	149.46	143.42
6	169.63	41.54
7	50.77	33.27
8	161.74	45.99
9	50.17	34.17
10	50.03	33.54
11	191.44	24.72
12	161.46	43.42
13	57.35	37.21
14	153.04	32.18
15	56.15	58.47
16	52.88	27.00
17	54.93	35.89
18	51.94	69.04
19	161.46	21.26
20	159.22	30.71
21	165.67	55.55
22	163.70	62.96
23	55.30	63.87
24	50.42	94.02
25	53.25	47.97
26	50.17	81.59
27	153.38	10.40
28	156.82	31.16
29	164.02	53.44
30	53.23	53.10
31	54.60	6.81
32	50.59	13.28
33	56.67	22.14
34	52.31	21.42
35	52.63	57.39
36	195.77	52.84
37	60.86	86.86
38	58.94	57.54
39	52.15	33.98
40	50.46	26.26

.....Contd....

Table 5.2 Contd...

well no.	transmissivity sq.m/day	storage (MCM)
41	50.29	52.54
42	423.94	13.94
43	428.69	31.52
44	403.36	44.20
45	71.46	46.09
46	50.41	31.51
47	51.41	43.15
48	53.04	65.65
49	53.87	71.72
50	54.25	91.78
51	210.35	71.17
52	50.84	36.24
53	54.25	25.91
54	71.46	36.17
55	220.85	29.81
56	232.81	55.04
57	51.95	59.40
58	169.74	39.47
59	58.26	50.34
60	57.60	69.09
61	51.03	74.90
62	71.77	63.10
63	51.01	93.17
64	50.33	67.27
65	169.74	177.42
66	50.22	72.60
67	54.57	23.53
68	53.24	40.07
69	54.72	54.45

estimated to be around 0.160. The angle between the principal permeability directions is estimated to be around 0.78 radians.

Storage values are estimated at all the 69 observation well points. For this purpose, one constant value for the thickness of the weathered and fractured zone is taken for each of the geological formations based on limited drill hole information. For granite and quartz arenites, this value was estimated as 35m from the surface while for limestones, this was fixed as 30m. The saturated thickness values (equal to the difference between the aquifer zone thickness and water table head) obtained at each of the observation points used for the estimation of storage. The estimates obtained for the two five-year block periods are indicated in Tables 5.2 and 5.5. The values vary from 10 to 140 MCM for granites which the same is in the range of 15 to 160 MCM for quartz arenites. The estimates for the limestone formations have worked out to be in the range 63 to 103 MCM. If information on the variation of thickness of weathered and fractured zones in the formations within the study area were available, the estimates of storage could have been more accurate. Further, it may be indicated here that although this storage is available around each well point, the extent to which the gorundwater exploitation can be carried out depends upon other management constraints.

The coefficients of recharge due to rainfall were also estimated at each observation point along with the

transmissivity and storativity values. For an uniform value of 0.15 given as input, the model output has indicated a spatial variation between 0.13 and 0.18 (Table 5.3).

The necessary software for the estimation of aquifer parameters in the present study was developed by the author. Algorithm for the same is prersented in Appendix B.

5.7 VALIDITY OF THE MODEL

The procedure of parameter estimation by optimization essentially involves arriving at such estimates of the aquifer parameters which consistently minimize the residues of the Boussinesq's equation, given by equation 5.12. These residues can be expressed in their simplest form as

$$E = \left[T_{xx} \frac{\partial^2 h}{\partial x^2} + T_{yy} \frac{\partial^2 h}{\partial y^2} \right] + Q - S \frac{\partial h}{\partial t} \qquad \text{.... (5.22)}$$

The three terms on the right hand side of the equation 5.23 represent the net excess inflow rate per unit area. Positive values of the second derivatives of the water table head indicate a concave surface and hence excess of inflows. This is represented by $\frac{\partial^2 h}{\partial x^2} > 0$ and $\theta_1 > \theta_2$, where θ_1 and θ_2 are angles between concave surface and its normal. Similarly negative second derivatives indicate a convex surface and the resultant excess outflow represented by $\frac{\partial^2 h}{\partial y^2} < 0$ and $\theta_1 < \theta_2$. A

Table 5.3 Estimated Rainfall Recharge Coefficient

well no.	coefficient	well no.	coefficient
1	0.1545	36	0.1324
2	0.1566	37	0.1589
3	0.1642	38	0.1652
4	0.1478	39	0.1520
5	0.1623	40	0.1753
6	0.1385	41	0.1528
7	0.1856	42	0.1577
8	0.1783	43	0.1735
9	0.1611	44	0.1712
10	0.1677	45	0.1521
11	0.1684	46	0.1687
12	0.1874	47	0.1745
13	0.1588	48	0.1824
14	0.1500	49	0.1750
15	0.1456	50	0.1648
16	0.1626	51	0.1686
17	0.1602	52	0.1618
18	0.1683	53	0.1539
19	0.1624	54	0.1605
20	0.1545	55	0.1470
21	0.1510	56	0.1547
22	0.1764	57	0.1726
23	0.1675	58	0.1663
24	0.1620	59	0.1539
25	0.1445	60	0.1824
26	0.1415	61	0.1510
27	0.1594	62	0.1574
28	0.1528	63	0.1538
29	0.1582	64	0.1825
30	0.1485	65	0.1543
31	0.1569	66	0.1652
32	0.1475	67	0.1615
33	0.1732	68	0.1707
34	0.1580	69	0.1586
35	0.1781		

zero second derivative indicates a plane surface with a complete balance between the horizontal inflows and outflows and the expression for that becomes $\frac{\partial^2 h}{\partial x^2} = 0$ and $\theta_1 = \theta_2$. The term Q represents net vertical accretion rate per unit area. The term $S \frac{\partial h}{\partial t}$ represents the storage increase per unit area. Thus, Q denotes the net imbalance between the total inflows (horizontal and vertical) and the change in groundwater storage. As per the continuity equation, this imbalance should be zero for all the time periods. However, the residues may never vanish completely because of inadequate quantification.

These residues are a function of T_{xx}, T_{yy}, S and other parameters representing the orientation of principal permeability directions and rainfall-recharge relations. Since these parameters are known or assumed to be time invariant, the objective is to arrive at such estimates of parameters for a specific space point, which consistently minimize as much as possible this imbalance at all the corresponding space points and for all the time periods. The minimization of the residue functional can be reviewed as multiple period water balance with time invariant parameters representing aquifer and hydrologic properties.

To acertain this, a second set of data (December 1982 to May 1988) pertaining to the water table heads, canal seepage, rainfall-recharge and well draft etc. are considered as input values to the model.The 66 months are divided in to 11 slabs as tabulated in Table 5.4.

Table 5.4 : Slabs for Parameter Estimation (Set 2)

sl.no.	period	slab no.
1.	Dec.'82 to May.'83	1
2.	Jun.'83 to Nov.'83	2
3.	Dec.'83 to May.'84	3
4.	Jun.'84 to Nov.'84	4
5.	Dec.'84 to May.'85	5
6.	Jun.'85 to Nov.'85	6
7.	Dec.'85 to May.'86	7
8.	Jun.'86 to Nov.'86	8
9.	Dec.'86 to May.'87	9
10.	Jun.'87 to Nov.'87	10
11.	Dec.'87 to May.'88	Overlap period

As already indicated the water table data for this 66 month period are also processed by least square polynomial approximation (Chapter 3) and the estimated derivatives are stored. The recharge components are also computed by the procedure explained in Chapter 4 and the generated values of recharge are also stored. Using all these inputs, the aquifer parameters are estimated for the second set by taking the same initial values and constraints as for the first set to test the sensitivity of the model. The estimated transmissivity values for this period are presented in Table 5.5. As can be seen from the results, the difference between the transmissivity values for first five year period (1978-83) and second five year period (1982-88) is minimal. As expected there is no variation in storativity estimates. Unfortunately the T and S values as obtained form pump tests are few and hence cross checking was possible only to a very limited extent.

The accuracy of the estimates was checked by simulating water table heads for the two five-year periods (1978 to 1983 and 1983 to 1988) using direct problem (Peaceman and Rachford, 1955). The procedure involved in this direct problem is presented in Appendix C. These simulated values of water table elevations are compared with collected observation well data.

In each of the two five year blocks, results estimated for one year duration each (June 1981 to May 1982 and December 1983 to November 1984) are presented as typical examples.

Table 5.5 Transmissivity and Storage Estimates (Set 2)

well no.	transmissivity sq.m/day	storage (MCM)
1	56.56	31.26
2	173.12	36.30
3	59.96	64.13
4	127.21	35.23
5	150.88	140.62
6	170.54	43.70
7	51.45	34.12
8	163.19	42.47
9	51.08	37.72
10	50.86	34.33
11	192.21	40.64
12	162.43	43.42
13	58.19	39.92
14	154.28	32.60
15	55.94	61.37
16	52.92	28.44
17	54.72	36.47
18	53.22	69.91
19	162.43	22.39
20	160.52	30.05
21	162.03	58.17
22	161.76	65.35
23	56.11	65.67
24	50.97	96.34
25	52.70	52.93
26	51.47	83.58
27	158.41	10.70
28	156.27	31.36
29	166.89	53.44
30	53.72	54.71
31	54.65	6.59
32	51.08	14.13
33	57.24	22.54
34	52.58	21.85
35	51.13	57.83
36	194.16	58.55
37	60.29	84.56
38	60.55	60.99
39	52.59	34.23
40	51.22	26.57

.....Contd.....

Table 5.5 Contd.....

well no.	transmissivity sq.m/day	storage (MCM)
41	50.63	54.53
42	424.28	15.17
43	430.85	35.34
44	408.68	42.14
45	72.60	54.29
46	51.86	31.96
47	50.99	44.64
48	55.00	70.21
49	54.16	74.79
50	54.79	92.31
51	215.09	68.39
52	50.88	38.84
53	53.65	26.20
54	71.15	37.95
55	221.35	30.58
56	238.10	57.83
57	51.86	59.95
58	162.72	38.23
59	58.95	50.25
60	58.34	63.72
61	52.09	83.72
62	72.11	63.79
63	52.17	107.80
64	50.23	73.62
65	160.23	162.93
66	50.69	73.47
67	53.45	30.26
68	54.29	42.75
69	54.68	58.81

Estimates for the period July 1981 (Table 5.6) indicated that the error varies between 0.001m and 9.724m. Out of 69 space points in each month about 18 points have indicated an error percentage of 5% and above. For each of the space points the error is represented as positive or negative sign depending on whether the observed value is higher or lower than the estimates. Results for one month (September, 1984) presented in Table 5.7 also revealed a similar trend of variation between the observed and simulated head data. In this period also the number of nodal points where the error is above 5% is similar. The pattern of errors is identical in both the cases for any space point (Tables 5.6 and 5.7).

The deviation of estimated values from the observed data for the groundwater heads can be due to several factors. The accuracy of these estimates to a certain extent can be improved by reducing the grid size as well as by reducing the time interval. This is true for the entire region for all the nodal points. However, the high values of errors observed at certain nodal points are predominantly due to the fact the effect of topographic, geological and land use factors are not completely taken into consideration in the recharge estimations. For example, the coefficient of rainfall recharge is chosen as 0.15 in the present study for the entire region, reflecting an overall average value. Due to non-availability of appropriate data, the spatial variation of this coefficient for different situations within the basin could not be

Table 5.6 : Results of Back Calculation (Set 1)

Period : July 1981

well no.	observed head(m)	predicted head(m)	error (%)
1	133.75	131.08	1.99
2	129.05	127.48	1.21
3	135.20	134.84	0.25
4	126.20	129.27	-2.43
5	132.60	138.01	-4.08
6	134.75	127.02	5.73
7	131.20	128.19	2.28
8	128.10	123.93	3.25
9	118.90	115.54	2.82
10	105.10	109.78	-4.45
11	109.15	107.18	1.79
12	133.35	134.74	-1.04
13	118.20	124.66	-5.46
14	118.84	120.07	-1.03
15	119.95	122.34	-1.99
16	119.10	118.99	0.08
17	110.10	113.05	-2.68
18	111.35	108.94	2.16
19	117.60	109.41	6.96
20	110.35	118.32	-7.22
21	138.05	128.79	6.70
22	120.40	125.62	-4.34
23	117.00	117.06	-0.05
24	103.70	110.63	-6.69
25	101.85	103.94	-2.06
26	110.90	103.24	6.89
27	93.20	100.93	-8.29
28	115.50	113.42	1.79
29	107.00	107.53	-0.49
30	109.88	110.92	-0.95
31	97.17	109.21	-12.39
32	106.60	103.51	2.89
33	114.80	108.51	5.47
34	107.50	104.38	2.89
35	114.05	112.69	1.18
36	102.95	106.85	-3.79
37	108.40	101.57	6.29
38	96.94	98.04	-1.13
39	107.70	98.52	8.52
40	101.10	104.38	-3.25

..... Contd.......

Table 5.7 : Results of Back Calculation (Set 2)
Period : September 1984

well no.	observed head(m)	predicted head(m)	error (%)
1	134.25	131.82	1.80
2	130.65	128.99	1.26
3	134.55	134.13	0.30
4	125.60	129.46	-3.08
5	131.50	136.75	-3.99
6	134.50	126.66	5.82
7	131.85	129.52	1.76
8	131.50	125.74	4.37
9	120.90	116.57	3.57
10	106.10	110.07	-3.74
11	110.85	107.98	2.58
12	132.70	133.97	-0.96
13	119.62	126.34	-5.61
14	121.94	121.83	0.08
15	120.90	124.23	-2.75
16	121.10	120.28	0.67
17	110.15	114.22	-3.69
18	110.35	110.10	0.22
19	120.70	113.29	6.13
20	111.15	119.22	-7.26
21	137.85	127.52	7.48
22	120.15	124.91	-3.96
23	117.17	118.58	-1.20
24	104.95	111.66	-6.39
25	103.85	105.06	-1.17
26	111.80	104.54	6.48
27	95.70	102.18	-6.77
28	114.90	113.54	1.17
29	105.80	108.08	-2.16
30	110.65	112.38	-1.57
31	101.90	110.70	-8.63
32	109.05	104.60	4.07
33	115.15	109.87	4.58
34	109.10	105.31	3.46
35	114.40	114.10	0.26
36	104.10	106.96	-2.74
37	107.15	102.13	4.67
38	98.50	99.03	-0.54
39	107.30	99.37	7.42
40	102.20	105.31	-3.05

......Contd......

Table 5.7 Contd...

well no.	observed head(m)	predicted head(m)	error (%)
41	103.00	109.67	-6.47
42	89.20	91.00	-2.02
43	95.50	93.78	1.79
44	106.90	101.40	5.14
45	91.20	90.36	0.91
46	100.00	92.85	7.14
47	86.90	93.64	-7.76
48	93.90	96.13	-2.37
49	100.90	102.18	-1.26
50	96.10	94.71	1.44
51	84.20	87.45	-3.86
52	90.20	90.25	-0.06
53	95.24	92.84	2.51
54	92.90	92.85	0.04
55	82.50	86.32	-4.63
56	75.45	83.07	-10.11
57	86.60	84.39	2.54
58	77.90	81.73	-4.92
59	94.20	86.32	8.36
60	88.70	80.75	8.96
61	87.20	83.11	4.68
62	75.80	81.08	-6.97
63	82.32	78.78	4.29
64	71.90	78.21	-8.77
65	72.55	72.73	-0.25
66	58.25	58.25	-0.00
67	62.20	63.90	-2.74
68	62.10	61.25	1.36
69	63.55	62.97	0.90

incorporated into the model. An analysis of data indicates that the nodal points with maximum errors are situated, in general, either closer to the canal areas or in the vicinity of dykes. For example the nodal points 31, 37 and 56 are closer to the dykes within the granite. Similarly the nodal points 27,60 and 67 are located close to lineaments observed on the satellite imageries. Nodal points 19, 20, 21, 27, 44, 47 and 58 are close to the canal system in the study region. Although, for nodal points such as the ones described so far the effect of a dyke or a canal or a weak zone appears to influence the deviation of the simulated head values, the cumulative effect of various parameters including land slope and land use is responsible for the observed errors.

To examine the influence of spatial variation of input data over the errors, split sampling technique has also been adopted. The coefficients of recharge and transmissivity estimates for various nodal points obtained as model ouput for the first five year block have been used as inputs for the respective nodal points for the simulation of water table head values for the second five year block. The deviation of the estimated head value from the observed head value at each of the observation wells has been calculated. The errors obtained through this approach for a period of one month (December 1983), are compared with the errors worked out on the basis of back calculation for the same period (Table 5.8). As can be seen from the Table, there is a significant reduction in the

Table 5.8 : Water Table Head Values (m) Estimated by Split Sampling

Period : December 1983

well no.	observed value	procedure A estimated	procedure A error(%)	procedure B estimated	procedure B error(%)
1	133.56	130.40	2.36	131.21	1.75
2	129.13	127.46	1.28	127.98	0.88
3	133.41	132.92	0.36	134.21	-0.60
4	124.98	128.33	-2.68	126.52	-1.23
5	130.15	135.75	-4.31	129.16	0.75
6	132.79	125.92	5.17	127.86	3.70
7	130.53	128.40	1.63	127.36	2.42
8	129.76	124.16	4.31	125.98	2.91
9	119.36	115.43	3.28	120.00	-0.53
10	105.43	108.95	-3.34	104.26	1.10
11	109.27	106.61	2.43	107.23	1.86
12	133.03	134.40	-1.03	135.16	-1.60
13	118.21	125.04	-5.78	123.77	-4.70
14	118.34	120.48	-1.81	121.43	-2.61
15	120.37	122.72	-1.95	121.98	-1.34
16	119.56	118.97	0.49	120.00	-0.37
17	109.54	112.97	-3.14	111.54	-1.83
18	109.68	108.78	0.81	109.88	-0.18
19	118.25	110.78	6.30	114.21	3.41
20	109.07	117.38	-7.62	115.65	-6.04
21	137.09	126.73	7.55	129.89	5.25
22	119.60	124.25	-3.89	118.36	1.02
23	116.63	117.43	-0.69	116.17	0.39
24	103.83	110.35	-6.28	105.35	-1.47
25	100.79	103.38	-2.57	99.98	0.79
26	110.42	102.75	6.94	103.77	6.01
27	94.25	100.66	-6.81	96.73	-2.63
28	114.67	112.2	2.10	115.28	-0.53
29	105.81	107.3	-1.48	106.43	-0.58
30	109.83	111.25	-1.29	112.78	-2.68
31	98.20	109.39	-11.4	103.77	-5.67
32	108.05	103.20	4.48	105.67	2.19
33	113.65	108.45	4.57	114.35	-0.61
34	109.20	103.85	4.89	104.47	4.32
35	114.28	112.60	1.46	113.74	0.46
36	103.00	106.06	-2.97	104.82	-1.76
37	106.45	101.44	4.70	103.55	2.71
38	97.20	97.77	-0.58	96.37	0.84
39	107.33	97.95	8.73	104.63	2.51
40	100.49	103.85	-3.34	99.67	0.81

.....Contd......

Table 5.8 Contd...

well no.	observed Value	procedure A estimated	procedure A error(%)	procedure B estimated	procedure B error(%)
41	100.60	109.20	-8.55	103.22	-2.61
42	88.40	90.30	-2.16	89.76	-1.54
43	95.52	93.08	2.54	94.51	1.05
44	104.05	100.71	3.20	103.9	0.13
45	92.41	90.05	2.54	93.65	-1.34
46	99.87	92.28	7.59	92.83	7.04
47	85.30	92.80	-8.79	88.27	-3.48
48	94.55	95.40	-0.90	93.82	0.76
49	101.75	101.83	-0.08	102.63	-0.87
50	95.90	94.44	1.51	95.22	0.70
51	82.84	85.77	-3.54	83.70	-1.04
52	90.01	89.70	0.33	90.46	-0.50
53	94.55	92.26	2.41	93.60	0.99
54	91.66	92.47	-0.88	91.38	0.29
55	82.60	85.66	-3.70	83.74	-1.38
56	75.66	82.14	-8.57	77.92	-2.99
57	85.91	83.46	2.84	86.92	-1.18
58	76.51	80.03	-4.60	78.89	-3.11
59	86.65	84.8	2.06	85.43	1.40
60	86.65	80.32	7.30	81.73	5.66
61	87.05	83.61	3.94	88.17	-1.29
62	75.10	80.33	-6.97	79.87	-6.35
63	79.61	77.23	2.98	78.74	1.08
64	74.50	78.00	-4.70	76.09	-2.14
65	71.95	70.75	1.65	72.45	-0.70
66	57.76	57.96	-0.34	57.82	-0.10
67	59.20	62.77	-6.04	58.63	0.95
68	60.33	59.94	0.63	58.98	2.22
69	64.12	61.64	3.86	62.51	2.50

(Procedure A : Back Calculation Method

Procedure B: Split Sampling Method)

error percentage when the split sampling method is adopted. With this method, the maximum error is around 7% and seven values were having an error above 5%. In contrast to this, the maximum error was around 11% and 18 values were above 5% when back calculation method was adopted.

CHAPTER 6

INTEGRATED PICTURE

The study area forms a part of the Nagarjuna Sagar left canal command area with three rivers (Musi river on west, Paleru river on east and Krishna river on south) and Nagarjuna Sagar left canal on the north as its boundaries. The basin covering about 830 sq.km is irrigated through the canals and groundwater for 9 months in a year.

The geological sequence is of granites, quartz arenites and limestones. The granites covering about 70% of the area are unclassified and belong to Archean age. Three small pockets of schists occur within the granitic terrain. In the western half of the granitic terrain, several dolerite dykes are present with a NE-SW trend. The granites are overlain by the quartz arenites which in turn are succeeded by the younger limestones both belonging to the Kurnool Super Group. These two sedimentary formations occur in the southern part of the basin. The lineaments and the faults located in the area, in general, are in conformity with the dykes in their trend, possibly coinciding with the regional tectonic trend.

For the groundwater evaluation, modelling techniques have been resorted to, based on the inverse problem as well as the

direct problem. The former was adopted for the estimation of aquifer parameters while the latter was used for the water table head estimation to check the validity of the model and also for simulating the changes in water table elevation for particular well draft and canal seepages.

The area has been divided into 20 x 19 grid (grid size 2.5km x 2.5km). The water table heads pertaining to the study area were collected at 69 observation points and for 120 months (June 1978 to May 1988). The data is divided into two blocks of 60 months each and each block is again divided into 10 slabs of 6 months period (June to November, December to May). To remove the noise present in the water table data and to find out spatial and temporal variation of the ground water head, the data were processed by least square polynomial approximation. In this approximation, a functional approach was adopted by minimizing the sum of squares of residual errors. As a first attempt, the coefficients of third degree polynomial were calculated and the minimum standard error for a typical example was estimated as 0.9012m and variance value as 0.8910 after deletion of 15 space points and three polynomial terms. As a next step to avoid loss of data a fourth degree polynomial was considered and the coefficients of the same were calculated with 13 terms and 63 space points with a standard error of 0.7243 and a variance of 0.9714.

It was observed that some space points in the region of dykes show anamalous values of groundwater head. An elevation

difference upto 15m has been evidenced for the water table heads at space points near dyke location. The polynomial coefficients generated by processing water table heads were used to calculate the second spatial derivatives and first temporal derivatives to help in aquifer parameter estimation. The software for processing water table data was developed by the author and the algorithm is presented in Appendix A.

Water table contour maps prepared on the basis of observed head values reduced to mean sea level have clearly indicated the groundwater table configuration. The flow is from north with gradient in all directions. In general, an increase in the spacing of the contours is evidenced in the vicinity of a dyke, lineament or a fault. The modification of the trend of contours within the study area is however due to a cumulative effect of all these structures as well as the canals which play an important role in their contribution to groundwater. The contour maps prepared indicating depth to water table from ground level have enabled a clear demarcation of water-logged localities in the study area. Trend surface maps also have been prepared to show the three-dimensional view of the groundwater table.

Monthly rainfall data collected at two stations in the study area have been assigned to different observation points in the model for computation of rainfall recharge at each of the observation pooints. For this purpose, a value of 0.15 has been taken for the coefficient of recharge due to rainfall.

Canal seepage values are calculated as ratio of the difference between discharges at upstream and downstream sides of a canal to the area of its wetted surface. The recharge due to canal seepage was computed using the prescribed norms as outlined earlier (Chapter 4, Section 4.6) and the values are assigned to all the 69 observation points and for 120 months. The variation of recharge due to canal seepage at different space points ranges for a typical month from 67×10^3 m^3 to around 11738×10^3 m^3 (Table 4.1).

Effective withdrawals from all existing wells of the study area were estimated for all 120 months using the available norms and these values are also assigned to each of the observation points.

The other components of groundwater balance equation such as evaporation and irrigation effect were also considered. Monthly evaporation data were obtained and a single value is assigned to all the space points. 5% of the canal discharge values were added to the evaporation data towards irrigation effect.

The inflows and outflows together with the net recharge values were calculated for all the months. Typical values for a month (March 1981) have been presented for all the 69 observation points (Table 4.1). The values of recharge vary between 45×10^3 m^3 to around 11715×10^3 m^3.

Using all the values of inflows and outflows and derivatives of water table head, estimation of aquifer

parameters has been carried out by discretizing the Boussineseq's equation and with the help of a non-linear optimization routine `Sequential unconstrained Minimization Technique', with chosen initial values and constraints. The software for the same was developed by the author and the algorithm for the same is presented in Appendix B. Storativity estimates were around 0.160, characteristic of the water table aquifers. While the transmissivity estimates in general were around 50 to 60 m^2/day, locations with dykes, lineaments and faults have registered higher transmissivity values. In addition to the aquifer parameters, the angle between the two principal permeability directions and coefficient of recharge due to rainfall were also estimated through inverse problem approach. The angle was estimated as 0.78 radians and the range of coefficient as 0.132 to 0.187. The storage values estimated at each of the observation wells ranged between 40 MCM to 180 MCM.

For checking the accuracy of the model, groundwater head values have been simulated using the estimated aquifer parameters and other related inputs, by adopting the Alternating Direction Implicit method. The estimated head values are then compared with the actual field values for all the 120 months. It has been observed that the difference between these two sets of values varied from 0.5% to around 11%. Of the 69 observation points, around 18 points were found to have an error above 5%. These anomalous values have been

attributed to the assumption in the calculation of recharge due to rainfall and to the effect of variation in geology, presence of dykes and land use. The errors in the estimation of water table heads could be further reduced significantly by assigning the transmissivity and the recharge coefficient estimates as obtained for all the 69 observation points at the end of the first five-year period, as inputs in the estimation of heads for the second block (1983-88). This split sampling procedure adopted had confirmed the effect of the spatial variation of the of the recharge due to geology and land use.

APPLICATION OF THE MODEL FOR THE STUDY OF WATER LOGGING

In the study area, since the inception of the canals the water table has registered an increase. In three regions, the water level is up to 3m below the ground level (Fig.6.1). In the region around Ponugodu in the NW part of the study area, the water table is about one metre from the ground resulting in water-logging conditions. To control this water-logging condition, a pilot study was conducted. This study involves introduction of a hypothetical drainage channel in the study area with assumed values of additional pumpage from the aquifer and simulating the aquifer behaviour by Alternating Direction Implict method (Peaceman and Rachford, 1955). This hypothetical channel has a length of about 16km and runs from observation point number 22 to observation point number 57 and

joins the canal number 17 (Mutyala). The assumed discharge of this unlined canal is 2.832 cumecs.

The data used for this study consists of the data at all space points comprising of the water table heads, storage factor, normal withdrawal rates and assumed additional draft values, transmissivity values and recharge factors etc. for the period December 1987 to May 88. The calculation of storage factor and value of error is similar as in the case of direct problem (Appendix C). The model is tested for five years and the head distribution at the end of May 1993 was estimated (Table 6.1). It is observed that with this assumed discharge values, the water table elevation in the study area would in general go down by about 1.0m at the end of five year period (May 1993). Particularly in the water-logged area (around observation point 21) in the vicinity of Ponugodu, decline in water table elevation would be as much as 3.00m. The contour plots for depth to water table for the month of December 1987 are presented in Fig.6.1 and the situation after introduction of the channel by May 1993 as simulated is shown in Fig.6.2.

Thus, in the present work, groundwater modelling has enabled evaluation of groundwater situation in the canal command area in terms of transmissivity and storativity. Prediction of groundwater heads has also been made for particular well drafts and canal flows. This model can thus be used for monitoring water-logged areas and to arrive at solutions for the same. It is however, suggested that the

Table 6.1 Comparision of Groundwater Levels (below ground surface) with Additional Pumpage

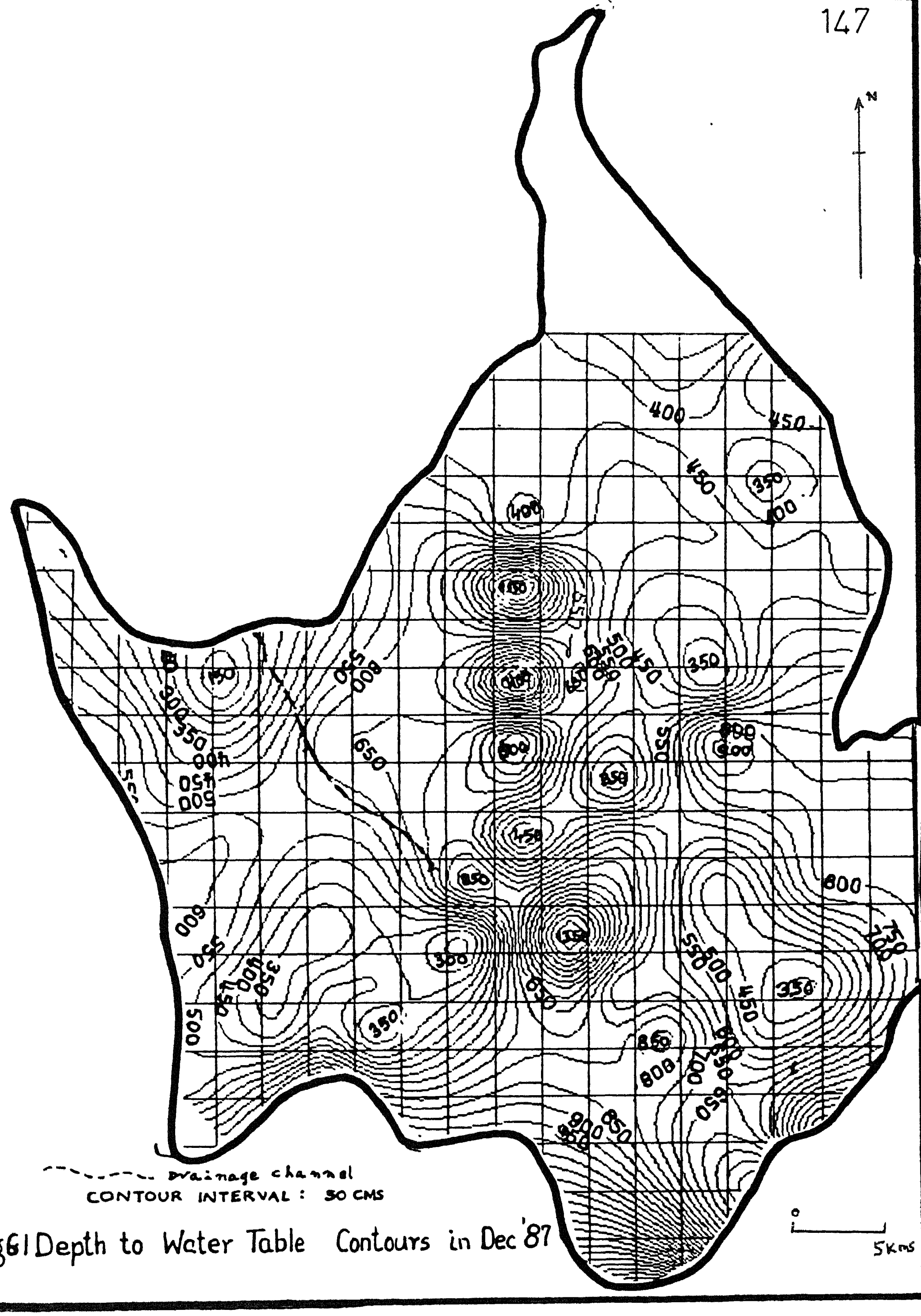
well no.	level in December'87	level in May'93
1	3.33	3.56
2	4.65	4.70
3	4.75	5.70
4	3.80	4.30
5	2.90	3.11
6	4.75	5.88
7	4.25	5.65
8	5.30	6.17
9	4.90	5.07
10	3.50	4.56
11	4.40	4.85
12	3.70	3.71
13	4.89	6.02
14	11.93	12.64
15	5.30	6.88
16	4.50	5.24
17	4.50	5.85
18	4.50	5.74
19	5.95	6.61
20	5.02	7.03
21	1.81	4.63
22	5.28	8.37
23	2.46	5.55
24	4.22	6.65
25	5.15	6.72
26	6.42	7.79
27	5.34	6.92
28	3.55	5.89
29	6.05	8.82
30	4.20	6.06
31	8.25	9.11
32	6.65	7.93
33	5.75	6.01
34	4.55	4.86
35	3.85	4.31
36	6.03	7.65
37	5.42	7.86
38	7.00	8.81
39	4.20	5.64
40	3.70	4.51

.....Contd......

Table 6.1 : Contd....

Well No.	level in December'87	level in May'93
41	8.85	9.65
42	5.35	6.74
43	3.40	4.10
44	4.94	5.84
45	8.00	10.28
46	4.20	7.61
47	8.00	9.25
48	4.75	6.16
49	6.65	7.52
50	8.10	9.47
51	8.20	9.29
52	4.40	5.87
53	4.50	5.61
54	3.10	3.49
55	4.57	6.77
56	3.56	4.24
57	3.38	4.30
58	6.90	4.12
59	12.70	11.57
60	4.10	5.05
61	10.50	9.32
62	4.20	4.67
63	8.60	8.77
64	4.50	5.06
65	8.40	8.75
66	11.15	11.46
67	18.25	17.11
68	7.15	7.71
69	4.25	4.38

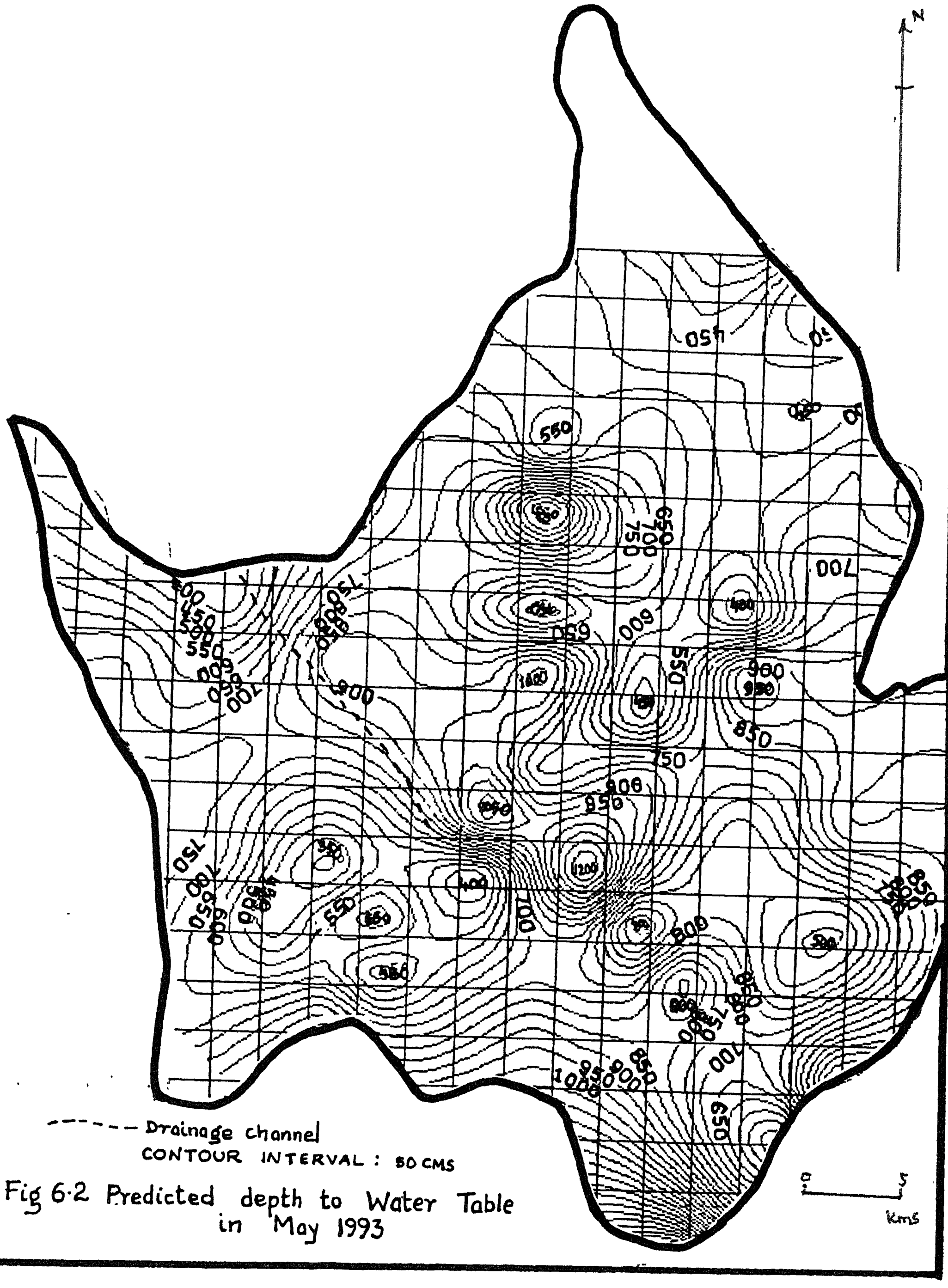
accuracy of the model can further be improved by taking into account the geology, land use, topography and field features while assigning the rainfall recharge inputs to the model. In addition, it is also suggested that the measurement of water table heads is to be carried out, in a field problem like this type, with specific care. It has to be ensured that when the water level in a well is measured, there is no interference of the same due to pumping at the adjacent sites.



- - - - - Drainage channel

CONTOUR INTERVAL : 50 CMS

Fig 61 Depth to Water Table Contours in Dec '87



---- Drainage channel

CONTOUR INTERVAL : 50 CMS

Fig 6·2 Predicted depth to Water Table in May 1993

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APPENDIX A

An algorithm has been developed to process the water table head values by least square polynomial approximation. The salient features of this algorithm to calculate the least square polynomials are presented below:

1. It normalizes the coordinates prior to the calculation of the coefficients of least square polynomials.

2. It detects and deletes the terms of the polynomials which do not contribute significantly towards expanding power of the polynomial. This is done on the basis of students 't' test.

3. It detects and deletes outliers based on the standard residue criterion.

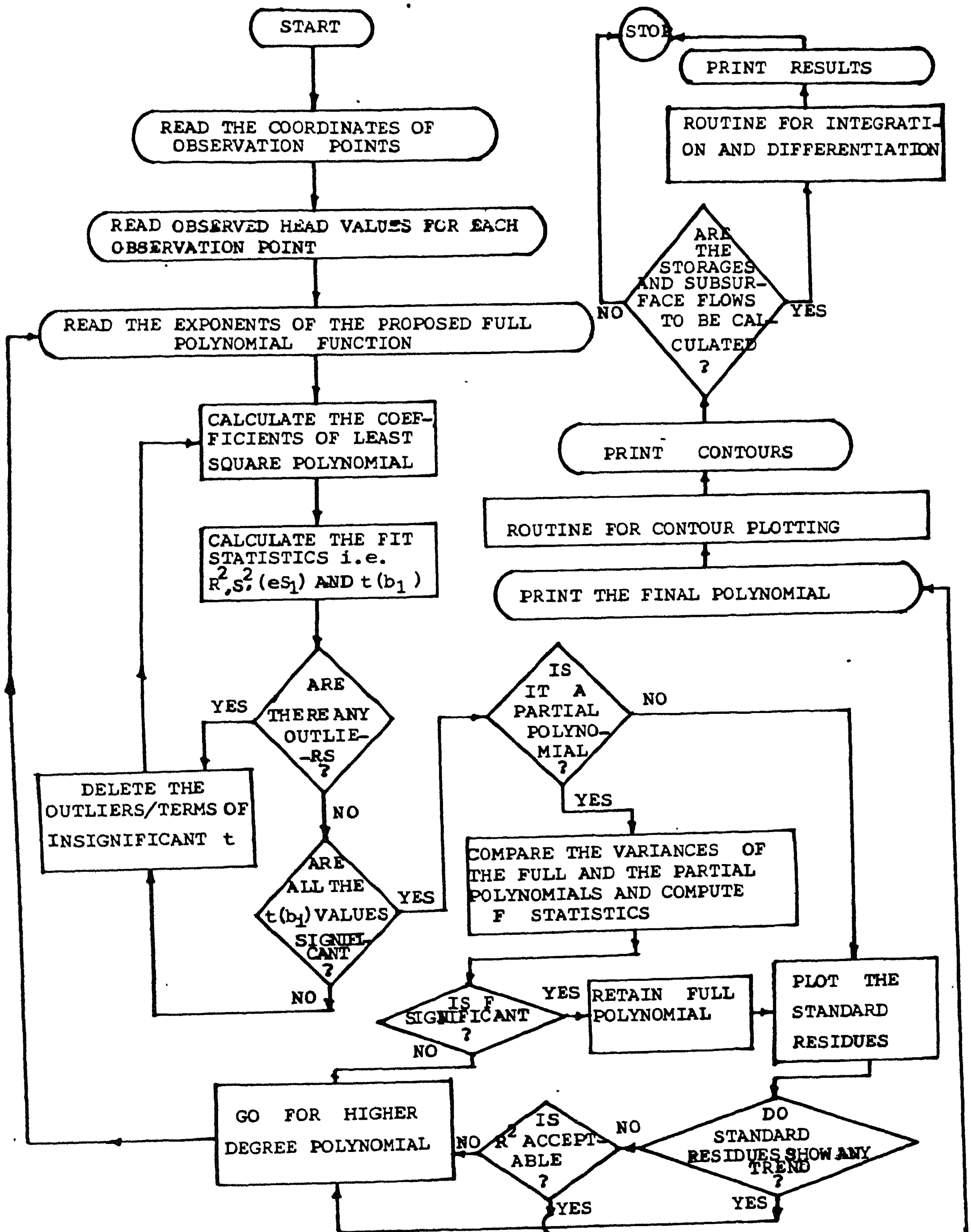
4. After deleting the terms and the outliers of the least square polynomial is re-computed with reduced number of terms and/or with reduced number of data points. This process is continued till the \`t' test does not warrant any further detection of the terms and all standard residues are acceptable.

5. After deciding the polynomial of minimum degree with minimum number of terms, corresponding least square coefficients are computed and out.

6. It also calculates, if desired mean water table elevation (by the process of integration) and mean boundary gradient (by the process of differentiation) corresponding to lateral flow and boundary recharge respectively.

7. The form of the least square polynomial and the coefficients are stored for further calculation.

The flowchart of the algorithm is presented in the next page.



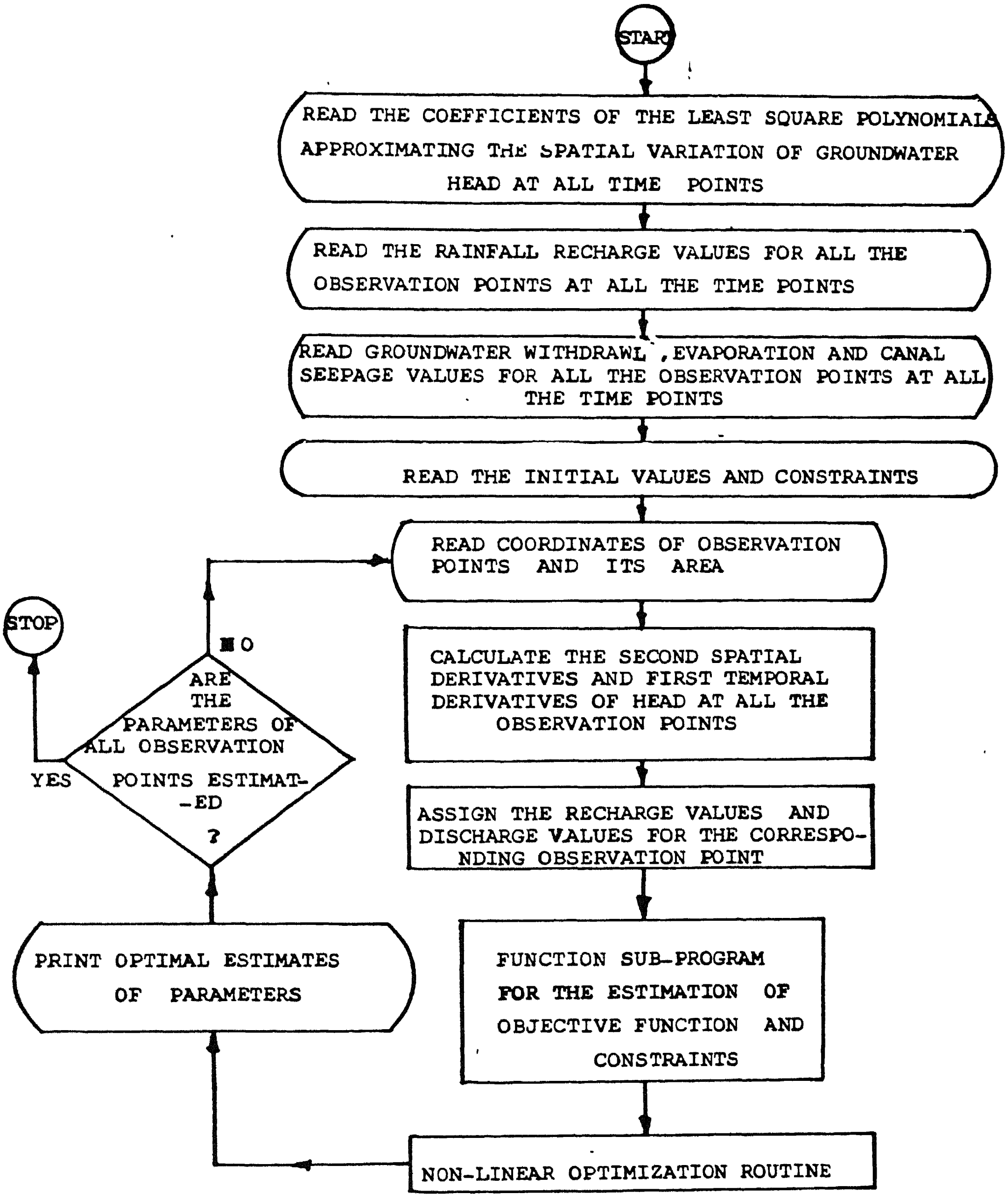
FLOW CHART FOR LEAST SQUARE POLYNOMIAL APPROXIMATION

APPENDIX B

The algorithm described in Appendix A provides the coefficients of polynomials at each well point. These coefficients help in the computation of spatial and temporal derivatives of the water table head at the space point under consideration (i.e. the space points for which the parameters are estimated) in each of the time periods. The following steps are involved.

1. It reads the coefficients of the least square polynomials.
2. It reads the rainfall recharge values at all the observation points and for all the time periods.
3. It reads the groundwater withdrawals, evaporation and canal seepage values for all space points and time periods.
4. It reads the initial values, constants, coordinates of the observation points and its area.
5. It calculates the second spatial derivatives and first temporal derivatives of head at all observation points.
6. It assigns the recharge and discharge values for the corresponding observation points.
7. An objective function is called through a sub-program and constraints and initial values are calculated.
8. The objective function inturn calls a nonlinear optimization routine (SUMT) to estimate optimal estimates of aquifer parameters.

A flow chart is presented in the next page indicating the algorithm used for the aquifer parameter estimation.



FLOW CHART FOR AQUIFER PARAMETER ESTIMATION

APPENDIX C

Estimation of water table heads from a given set of data, such as transmissivity, storage factor, withdrawal rates etc., is called as direct problem in groundwater hydrology. Prickett and Lonnquist (1971) developed an algorithm for prediction of water table heads using finite difference method. In their algorithm they incorporated Iterative Alternating Direction Implicit method (Peaceman and Rachford, 1955). It involves first, for a given time increment, reducing a large set of simultaneous equations down to a number of small sets. This is done by solving the node equations by Gauss elimination by keeping individual columns/rows constant. The set of column/row equation is then implicit in the direction along the column/row and explicit in the direction orthogonal to the column/row alignment.

The process of calculating heads along columns or rows in the digital model includes first computing the values of G and B as explained by Prickett and Lonnquist (1971ü for the nodes of a column or row in increasing order. In the course of accomplishing this, the head at the last node of the column or row is found. After completing the calculation of heads in an individual column or row, the computer proceeds to the next column or row until all have been processed satisfactorily.

In the present study, after estimating transmissivity and storativity by inverse problem the same are cross checked by direct problem. Direct problem has also been used to predict water table heads after introducing a hypothetical drainage channel in the study area.

A 20 X 19 grid was considered for the simulation of heads. The input data consists of heads at start of time increment, storage factor, withdrawals, transmissivity, number of time increments etc.. Storage factor (SF_1) is calculated as

$$SF_1 = 7.48 * S * \Delta x * \Delta y \qquad \text{...... (A.1)}$$

where S = storativity,

$\Delta x, \Delta y$ = grid interval in x, y directions

The error is estimated as

$$\text{Error} = Q * \text{DELTA} / 10 * \text{SF1} \qquad \text{...... (A.2)}$$

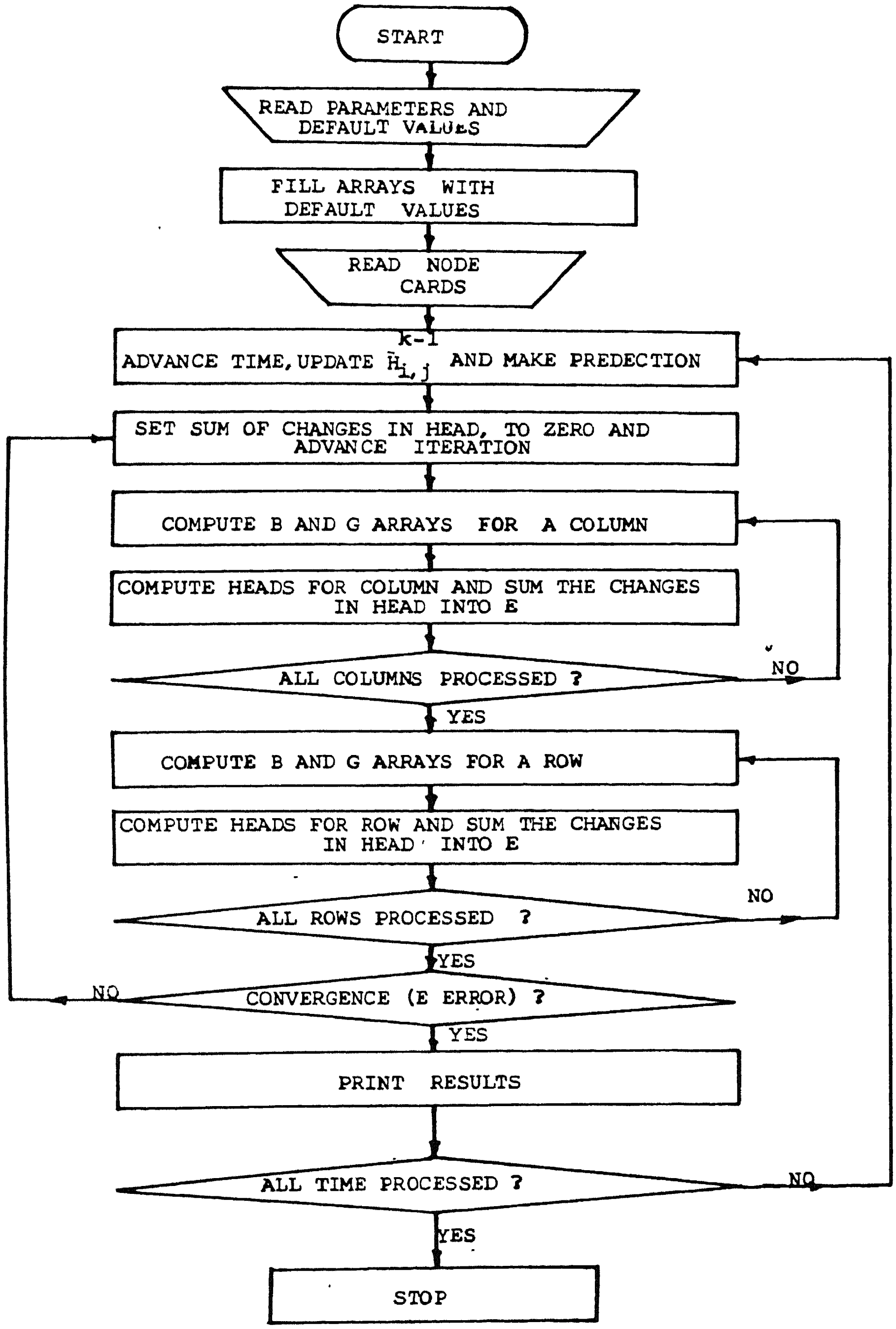
where Q = withdrawals,

$$\text{DELTA} = \text{time}/\ \delta \qquad \text{......... (A.3)}$$

where δ is a function of time step.

The process associated with this algorithm is presented in the next page. The program can do the following operations:

1. It reads parameter and default value cards for all the space points.
2. After reading node cards it makes prediction of heads.



FLOW CHART FOR DIRECT PROBLEM (AFTER PRICKETT AND LONNQUIST 1971)

3. With advance in time, it computes B and G arrays for a column and make changes in heads.

4. After processing all the columns, B and G arrays are estimated with respect to a row and changes are made in heads.

5. When heads are re-estimated for all the rows also it checks for convergence.

6. When the convergence is satisfied it prints predicted head values for this time increment.

7. The above steps are repeated till the end of time intervals.